Competing with Menus of Tariff Options*

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Abstract

The nature of numerous strategies of firms is often discrete or countable. This adds difficulty to measuring and testing for the existence of complementarities among several strategies. This paper introduces a generalized multivariate count data model that allows estimating correlations of any sign among the pricing decisions of competing firms in a manner that is robust to the existence of unobserved heterogeneity leading to either over and underdispersion of the distribution of counts. Thus, it is possible to overcome a major challange in testing whether two decisions are strategic complements or substitutes, *i.e.*, dealing with the effect of unobserved heterogeneity. I study how firms actually compete in nonlinear tariffs by analyzing the interrelation between the incumbent and entrant's decisions to offer a given number of tariff options. Results document the existence of complementarity among tariff options regardless of whether they are dominated or not. This result supports the view that the implementation of nonlinear tariffs by means of a menu of self-selecting two-part tariffs has some strategic value in competitive environments.

Keywords: Competitive Tariff Menus; Strategic Complementarity; Bivariate Count Data Regression; Double Poisson; Copula Functions.

JEL Codes: C35, D43, M21

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1 Introduction

The nature of numerous strategies of firms is often discrete or countable. Firms either enter or not in a given market. Chains decide how many stores to open in each location. Firms also decide whether to innovate or not, whether to acquire another company or not and how many products to commercialize. The framework popularized by Milgrom and Roberts (1990a) and Topkis (1998) is capable of addressing environments where firms' strategies are discrete and their decision problems fail to be convex. The combination of supermodular functions defined on lattices has become the standard tool to address the existence of strategic complementarity among firms' decision. However, this non-continuous nature of firms' decisions adds significant difficulty to measuring and testing for the existence of strategic complementarities.

Take for instance the case of nonlinear pricing. Firms engaging in nonlinear pricing rarely make use of fully nonlinear tariffs. Actually, firms only offer few tariff options that approximate the fully nonlinear tariff. Thus, their pricing strategy consists of a bundle of countable features that characterizes each tariff option. A tariff option can therefore be interpreted as defined on a lattice of tariff features. In addition, when firms compete, their pricing tactics may have important strategic effects: offering more or less tariff options may induce consumers to switch carriers or different types of customers to subscribe to each company. Thus, a menu of numerous tariff options by one carrier may trigger a similar response by the competitor if the offering of tariff options are strategic complements, or a smaller number of options if they are strategic substitutes.[1] In this latter case, competing firms may differentiate their otherwise (almost) identical products through tariff design by appealing to customers that are significantly heterogeneous regarding the different features of the tariff options, *e.g.*, such as their complexity.

The present paper analyzes the pricing strategies of competing carriers in the early U.S. cellular telephone industry between 1984 and 1992. Table 1 cross-tabulates the occurrences of the different strategies. Firms offered only few tariff options as a practical way of implementing a screening mechanism among heterogeneous customer bases. In addition, I distinguish between the number of *actual* and *effective* tariff plans. An effective tariff plan is the least expensive option for some non-negligible usage pattern. Effective tariff plans define the lower envelope of the tariff while the rest are *dominated* in the sense that it is always possible to find a least expensive tariff for each possible usage pattern.

Table 1 is the first piece of evidence in favor of strategic complementarities in the number of tariff options. Regardless of whether we focus on the actual or on the effective number of tariff options, it is very infrequent to observe one firm offering only one or two options while the other attempts to segment the market by flooding customers with numerous choices. Most of the time, firms offer between 2 and 4 options

[1] See Milgrom and Roberts (1990b) and Vives (1990) for a theoretical treatment of strategic complementarities when firms' strategies are not continuous.

Table 1: Contemporaneous Correlation Among Number of Tariff Options

	Actual Tariffs							*Effective Tariffs*						
Plans	1	2	3	4	5	6	All	1	2	3	4	5	6	All
1	1	0	1	0	0	0	2	5	0	1	1	1	0	10
2	4	6	5	2	1	0	18	1	10	8	4	1	0	17
3	0	1	10	6	3	2	22	0	4	12	5	3	1	33
4	0	3	6	11	6	2	28	3	2	9	7	3	0	23
5	1	1	3	9	9	1	24	1	1	3	6	6	1	14
6	0	0	1	3	0	1	5	0	0	0	0	0	0	2
All	6	11	26	31	19	6	99	10	17	33	23	14	2	99
Kendall's τ	0.3668 (5.38)							0.3280 (4.81)						

Total cases for each combination of tariff options offered by the incumbent and entrant firm. Rows indicate the number of options of the entrant and columns those of the incumbent. Kendall's τ measures the association among the number of tariff options. The corresponding absolute value t-statistics are shown in parentheses. There are 99 pairs of tariff strategies in the sample.

while the competitor offers a similar number, differing at most in one option. For both type of tariffs, the unconditional correlation among the number of tariff plans offered is positive and significant.

Evidently, this approach to test for the existence of complementarity has serious shortcomings. Two firms compete in each city. Cellular services are essentially identical across firms and they likely face the same potential customer base. If commercialization costs are also similar across firms, it is likely that they end up offering the same number of tariffs. Thus, the number of tariff plans would lack any strategic value and it might well be that once we condition on observable firm and market characteristics the strategic complementarity effect disappears. Furthermore, whether consumers have a biased taste in favor of one of the carriers, or whether commercialization costs are asymmetrically distributed across carriers is unobservable to the econometrician. Athey and Stern (1998) first pointed out the importance of controlling for unobserved heterogeneity when testing for the existence of complementarities as the unconditional correlation among strategies may well be just the result of unobserved heterogeneity common to the different firms in an industry. Miravete and Pernías (2006) first tested for the existence of *complementarity* among different kind of innovation strategies in a pooled sample of Spanish firms while controlling for the existence of unobserved heterogeneity.

Alternatively, this paper focuses on the existence of *strategic complementarity* among the pricing tactics of competing firms while controlling for the existence of observed and unobserved heterogeneity. In doing so, the paper presents an econometric estimation method that overcomes the difficult issue of measuring correlations among count variables. Furthermore, the econometric model is able to accommodate both an overdispersed and underdispersed distribution of counts, which commonly arise as the result of the existence of unobserved heterogeneity, *e.g.*, Cameron and Trivedi (1998, §4).

The econometric model of this paper estimates a pseudo-maximum likelihood procedure based on a Gaussian copula function with double Poisson marginals. This approach uniquely accommodates the observed underdispersion of the distribution of the number of tariffs and the possibility of negative correlation among the distribution of tariff options of the incumbent and entrant in the early U.S. cellular telephone industry. It should be noted that this econometric model is the first that allows econometricians controlling for any combination of over and underdispersed distribution of counts together with the possibility of negative correlation among the number of tariff options offered by the competing firms.[2] This is a sensible economic hypothesis that cannot be ruled out by imposing a positive correlation leading necessarily to conclude that the number of tariff options offered by the incumbent is a strategic complement to the number of tariff options offered by the entrant and *vice versa*. Furthermore, and contrary to the few existing multivariate count data models, the consistent estimates are robust to the existence of unobserved heterogeneity leading to over and underdispersion of counts and the estimation approach, which avoids numerical integration, proves to be fast and easy to implement.

Results support the view that strategic complementarity among tariff options are important regardless of whether they belong to the lower envelope of the tariff function or not. Since these results already control for the existence of observed and unobserved heterogeneity, we can conclude that the number of tariff options has some strategic value and that firms will rather imitate the pricing practices in the industry than use pricing as a way to differentiate from competitors. Thus, results are more in line with the symmetric equilibrium models of nonlinear pricing of Armstrong and Vickers (2001) and Rochet and Stole (2002) than with the business stealing through pricing model of Yang and Ye (2008).

The paper is organized as follows. Section 2 briefly describes the data. Section 3 presents a feasible multivariate count data regression model based on the Gaussian copula and double Poisson distribution marginals. Section 4 reports the results of this bivariate count data regression model in which the number of total and non-dominated tariff options offered by each firm is regressed against firm characteristics and time and market fixed effects. This section also suggests interpretations that are consistent with the reported results and evaluates the existence of strategic complementarities among firms' offerings of tariff plans. Section 5 concludes.

2 Pricing in the Early U.S. Cellular Industry

In this paper I study the number of tariff options offered by the competing firms of the largest U.S. cellular telephone markets between 1984 and 1992. I ignore other tariff features such as length of the contract,

[2] Miravete (2008) recently developed two alternative, constrained, maximum likelihood estimators that can also address these uncommon features of the distributions of counts by making use of the multivariate Sarmanov distribution.

Table 2: Descriptive Statistics

	Incumbent		*Entrant*		*All Firms*	
Variables	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
PLANS	3.6970	1.2244	3.6465	1.2561	3.6717	1.2374
EFFPLANS	3.2020	1.2287	3.2020	1.2370	3.2020	1.2297
LASTMONO	1.5152	1.9025	1.5152	1.9025	1.5152	1.8977
LSTEFMON	1.1515	1.4240	1.1515	1.4240	1.1515	1.4204
YEAR88	0.3333	0.4738	0.3333	0.4738	0.3333	0.4726
YEAR92	0.3333	0.4738	0.3333	0.4738	0.3333	0.4726
FIRM-AGE	45.3030	31.5919	34.6432	30.5769	39.9731	31.4666
BELL	0.6667	0.4738	0.3030	0.4619	0.4848	0.5010
AP_{peak}	0.2288	0.2606	0.2210	0.2812	0.2249	0.2705
$AP_{off-peak}$	0.1526	9.5539	-1.4083	11.8900	-0.6279	10.7864
COVERAGE	0.0964	0.0789	0.0964	0.0789	0.0964	0.0787
Observations	99		99		198	

All variables are defined in main text.

bundling with particular telephone sets, or magnitude of the allowance of each tariff plan. These features certainly have an effect on the decision to subscribe to a particular plan and/or carrier, but the information on most of them is quite incomplete. Thus, I will focus on the number of tariff plans only and presume that there is significant unobserved heterogeneity that needs to be dealt with at the econometric stage.

Table 2 presents descriptive statistics by type of firm. The data set, which has been described at length elsewhere,[3] contains a complete description of the tariff options offered by any of the two firms present in the largest markets of the U.S. between 1984 and 1988. This information was collected by *Economic and Management Consultants International, Inc.* and reported in *Cellular Price and Marketing Letter*, Information Enterprises, various issues, 1984–1988. For year 1992, Marciano (2000) combined information from *Cellular Directions, Inc.*, the *Cellular Telephone Industry Association*, and direct interviews with managers. Some of the tariff options offered, PLANS, were always more expensive than others, and in this sense they are considered to be *dominated*. The effective tariff options, EFFPLANS, are those that are the least expensive option for at least one out of the half a million usage profiles defined by the sum of peak and off-peak minutes adding up to a maximum of 1000 minutes of usage a month.

Table 3 presents the histogram of the actual and effective number of tariff options by type of cellular carrier. Firms in this early market offered fewer tariff options than what is thought of being customary today. It is noticeable though that while the number of effective tariff options is always smaller than the actual number of options, the difference is not excessive. Another remarkable feature of the data is that the distribution of plans is always underdispersed, *i.e.*, the variance of the distribution of number of plans never exceeds the mean, which is the opposite of what is normally encountered in dealing with count data.

[3] See for instance Busse (2000), Miravete and Röller (2004), and Parker and Röller (1997).

Table 3: Frequency Distributions of Number of Actual and Effective Tariff Options

	Actual Options				*Effective Options*			
	Incumbent		*Entrant*		*Incumbent*		*Entrant*	
Actual Options	Cases	Rel.Freq.	Cases	Rel.Freq.	Cases	Rel.Freq.	Cases	Rel.Freq.
1	2	0.0202	6	0.0606	8	0.0808	10	0.1010
2	18	0.1818	11	0.1111	24	0.2424	17	0.1717
3	22	0.2222	26	0.2626	25	0.2525	33	0.3333
4	28	0.2828	31	0.3131	24	0.2424	23	0.2323
5	24	0.2424	19	0.1919	18	0.1818	14	0.1414
6	5	0.0505	6	0.0606	0	0.0000	2	0.0202
Mean, (Var.)	3.6969	(1.4991)	3.6464	(1.5778)	3.2020	(1.5098)	3.2020	(1.5302)

Absolute and relative frequency distributions of the number of actual and non-dominated tariff options offered by each active firm.

In addition I use some variables to control for the effect of time and market heterogeneity. Every firm is observed three times in this data set: at the earliest quarter of the duopoly phase of the market between 1984 and 1988, in the third quarter of 1988 (YEAR88), and in 1992 (YEAR92), respectively. Depending on the time of the award of the *wireline* license to the incumbent firm and court resolutions (market by market) over disputed *non-wireline* licenses awarded to one of the many potential entrants, each firm accumulates a different experience, measured by FIRM-AGE, the number of months present in a market at the time of each observation. In addition to all these time related variables, BELL indicates whether the largest shareholder of each firm used to be one of the "Baby Bells" (information available from the *Federal Communications Commission*), which in principle may have some influence on pricing practices and/or ability to design plans for these firms. Furthermore, LASTMONO and LSTEFMON are the number of actual and effective tariff options, respectively, offered by the incumbent during the last quarter during which it enjoyed its monopoly position. Since the monopoly pricing strategy was known, it could reveal relevant information about the heterogeneity of consumers to the entrant, therefore influencing the future pricing decisions of both the incumbent and the entrant. All variables are time invariant except FIRM-AGE, AP_{peak}, and $\text{AP}_{\text{off-peak}}$.

In any nonlinear pricing problem, a major issue is to characterize the nature and magnitude of asymmetric information regarding consumers' valuation of the product or service. I take an equilibrium approach and assume that tariff plans are designed optimally to screen consumers provided some fixed cost of commercialization per tariff that limits the actual number of them offered to consummers. Firms know the distribution of consumer types and its features critically condition the curvature and position of the optimal fully nonlinear tariff. Thus, I include as regressors the curvature of the peak and off-peak tariff schedule as defined by AP_{peak} and $\text{AP}_{\text{off-peak}}$, and the ratio of total to potential subscribers, COVERAGE. Variable AP_{peak} is the equivalent of the Arrow-Pratt measure of risk aversion computed over a thousand minute interval of airtime usage by means of the quadratic polynomial that best fits the lower envelope of the peak component of the tariff. Variable $\text{AP}_{\text{off-peak}}$ is similarly computed using the off-peak component of the tariff instead. The idea behind these measures of risk aversion is that the more heterogeneous consumers are, in the sense that the distribution of their tastes has a higher hazard rate, the more concave the optimal tariff needs to be. If all consumers are alike, a single two-part tariff suffices to extract all the rents from consumers.[4] Evidently, the degree of concavity computed makes use of the actual tariffs offered, and thus, these two variables may be endogenous. Finally, COVERAGE is defined as the maximum capacity installed (measured by the number of antennas × 1,300 customers) divided by the number of businesses in a particular city and the population divided by four (to approximate the number of households). This variable may also be endogenous as the participation decision of consumers may depend on tariff options tailored to their tastes. Thus, more numerous and less expensive options could lead to significantly higher participation rates.

3 A Bivariate Double Poisson Count Data Regression Model

Regardless of whether we focus on the total number of tariff options offered by the competing firms or on the number of effective tariff options, Tables 1 and 3 present us with two features that none of the existing count data regression models can handle simultaneously: the distribution of counts appears to be underdispersed and positively correlated across firms. The bivariate double Poisson count data regression model is a flexible approach based on a Gaussian copula function with assumed double Poisson marginal distributions and correlation of unrestricted sign that can be easily implemented and allows us to address simultaneously over or underdispersion and a potentially negative correlation of counts, a case that is excluded in the few multivariate count data models available.

In presenting the econometric model, I will consider first the single equation count data regression model behind the assumed double Poisson marginal distributions of the bivariate model. Let $y_i = 0, 1, 2, \ldots$ be distributed according to a double Poisson distribution with parameters μ_i and θ_i, conditional on a set of regressors $\boldsymbol{x_i}$ in a sample with $j = 1, 2, \ldots, n$ observations. Efron (1986) shows that the probability frequency function of this double Poisson is:

$$f_i(y_i, \mu_i, \theta_i | \boldsymbol{x_i}) = K(\mu_i, \theta_i) \sqrt{\theta_i} \exp(-\theta_i \mu_i) \exp(-y_i) \frac{{y_i}^{y_i}}{y_i!} \left(\frac{\exp(1)\mu_i}{y_i} \right)^{\theta_i y_i}, \tag{1a}$$

$$\mu_i = \exp(\boldsymbol{x_i}' \boldsymbol{\beta_i}), \tag{1b}$$

$$\frac{1}{K(\mu_i, \theta_i)} \simeq 1 + \frac{1 - \theta_i}{12 \theta_i \mu_i} \left(1 + \frac{1}{\theta_i \mu_i} \right). \tag{1c}$$

[4] This approach is similar to the discrete Arrow-Pratt measure of Marciano (2000, §4.2) and the Cobb-Douglas approximation to the lower envelope of the tariff of Busse and Rysman (2005).

The advantage of the double Poisson over the standard Poisson distribution is that the mean and variance do not depend on a single parameter. Efron (1986) shows, among many other results, that:

$$\mathrm{E}[y_i|\boldsymbol{x_i}] \simeq \mu_i\,, \tag{2a}$$

$$\mathrm{V}[y_i|\boldsymbol{x_i}] \simeq \frac{\mu_i}{\theta_i}\,. \tag{2b}$$

Hence, the double Poisson includes the standard Poisson as a particular case, when $\theta_i = 1$, but it allows for overdispersion when $\theta_i < 1$ as well as for underdispersion if $\theta_i > 1$.[5] Furthermore, the maximum likelihood first order conditions of the double Poisson coincide with those of the standard Poisson regression model. The estimation of the parameters of interest $(\boldsymbol{\beta_i'}, \theta_i)'$ then proceeds in two steps:[6] First, a simple and fast iteratively reweighted least squares procedure (Cameron and Trivedi (1998, §3.8)) is used to estimate the components of $\boldsymbol{\beta_i}$, and through equation (1b), $\hat{\mu}_{ij}$. The estimate of θ_i is obtained as the maximum likelihood estimator after substituting $\hat{\mu}_{ij}$ into the probability frequency function (1a), and it coincides with the sample mean of the deviance measure (Cameron and Trivedi (1998, §5.3.2)):

$$\hat{\theta}_i = \frac{1}{n}\sum_{j=1}^{n} y_{ij} \ln\left(\frac{y_{ij}}{\hat{\mu}_{ij}}\right) - \left(y_{ij} - \hat{\mu}_{ij}\right)\,. \tag{3}$$

The present copula-based model extends the approach of Efron (1986) to deal with multivariate counts making use of a Gaussian copula function. The clear advantage of this approach is that it is easily implementable. Estimation is fast and easy and it only involves few computations beyond those of a pseudo-maximum likelihood count data model in order to control for over/underdispersion and correlation among counts. This is in stark contrast with the existing literature on multivariate counts which normally requires heavy numerical integrations. The bivariate double Poisson count data regression model can furthermore accommodate any kind of correlation among counts. Furthermore, these two features of the joint distribution of counts are not driven by a common unobserved factor to all univariate marginal distributions and the parameterization of the likelihood function allows for all possible combinations of over or underdispersed marginals and correlation of any sign. Allowing for the estimation to be flexible enough to accommodate the observed dispersion pattern in the data reduces the risk of misspecification

[5] It is well known that the existence of individual unobservable heterogeneity may explain the violation of the equidispersion property of the Poisson distribution. Indeed, Greenwood and Yule (1920) first derived the popular negative binomial alternative to the Poisson distribution as a mixture of a Poisson process with a gamma distributed multiplicative heterogeneity component. See Cameron and Trivedi (1998, §4.2) and Winkelmann (2008, §3.3.2) for a complete overview of this topic. However, the cross-section negative binomial regression or the corresponding panel data specification of Hausman, Hall, and Griliches (1984) only address successfully the case of overdispersion rather than the underdispersion encountered in the present data.

[6] This result holds when, as in the case considered here, θ_i is assumed to be a constant because in such a case the gradient characterizing the first order conditions of the maximum likelihood estimation is a block recursive system of equations where θ_i does not enter into the equations determining the components of $\boldsymbol{\beta_i}$.

bias. This is also accomplished by not excluding the possibility of negative correlation between the counts. Furthermore, extensions of this model to multivariate cases is straightforward.

However, there remains an obstacle that needs to be addressed in order to apply the Gaussian copula to a joint distribution of counts. Under general conditions copulas are uniquely defined as long as the marginal distributions are continuous. This is not the case for count data variables because they are defined on $\mathbb{N}$ rather than on $\mathbb{R}$. Denuit and Lambert (2005) provide the solution to this problem by generating a *"continued"* variable $y_i^\star$ for each observable discrete realization of the count, y_i. The idea is to add an independent random draw u from a standard uniform distribution $U[0,1]$ to each count y_i so that the newly generated continuous distributions preserve the concordance order of the original discrete distributions, *i.e.*, the association between the counts. Thus, a count data y_i is continued by $U[0,1]$ as follows:

$$y_i^\star = y_i + (u-1)\,, \tag{4}$$

so that $y_i^\star < y_i$ almost surely. Thus, for practical purposes fixing the problem of the discrete nature of the endogenous variable only adds the simple step to computing φ_i as the probability integral transformation of $y_i^\star$, the continued variable associated with y_i. Without loss of generality, let $F_i^\star(y_i^\star)$ denote the continuous marginal distribution function of the continued variable $y_i^\star$. The distribution of $y_i^\star$ is just the convolution distribution of y_i and u:

$$\varphi_i = F_i^\star(y_i^\star) = F_i(y_i + (u-1)) = F_i(y_i - 1) + f_i(y_i)\cdot u\,, \tag{5}$$

where in the present model $F_i(\cdot)$ and $f_i(\cdot)$ correspond, respectively, to the discrete probability distribution and density functions of the double Poisson of equation (1a). Noticing that $dy_i^\star = du$, equation (5) leads to another important result for the estimation of this model:

$$f_i^\star(y_i^\star) = f_i(y_i)\,. \tag{6}$$

Next, I need to analyze how the marginal distributions of two continued variables are combined into a joint bivariate distribution. Let $(y_1^\star, y_2^\star)$ be two continuous variables with joint probability density function $F^\star(y_1^\star, y_2^\star)$ and with continuous univariate marginal probability distribution functions given by $F_i^\star(y_i^\star)$, $i = 1, 2$. The basic result on copulas, first proven by Sklar (1959), states that the following copula function $C^\star$ exists and is unique:

$$C^\star\colon [0,1]\times[0,1] \Rightarrow [0,1], \quad \text{s.t.} \quad F^\star(y_1^\star, y_2^\star) = C^\star\left(F_1^\star(y_1^\star), F_2^\star(y_2^\star)\right)\,, \tag{7}$$

or alternatively, if $C^\star(\cdot)$ is a copula, then $F^\star(\cdot)$ is a joint probability density function with marginal distributions $F_i^\star(\cdot)$. Thus, a bivariate copula with standard uniform marginals can always be written as follows:

$$C^\star(\varphi_1, \varphi_2) = F^\star\left(F_1^{\star-1}(\varphi_1), F_2^{\star-1}(\varphi_2)\right) , \tag{8}$$

for $\varphi_i = F_i^\star(y_i^\star)$, $i = 1, 2$, as indicated in equation (5), because φ_i defined in this manner is always uniformly distributed when the marginal distribution functions $F_i^\star(y_i^\star)$, $i = 1, 2$ are continuous. The Gaussian copula simply assumes a standard joint bivariate normal distribution with standard univariate normal marginal distribution functions in equation (8). This allows me to augment the copula function with parameter ρ to account for any correlation among the continued variables:

$$C_\Phi^\star(\varphi_1, \varphi_2; \rho) = \Phi_{[2]}\left(\Phi_{[1]}^{-1}(\varphi_1), \Phi_{[1]}^{-1}(\varphi_2); \rho\right) , \tag{9}$$

where $\Phi_{[\kappa]}(\cdot)$ denotes the κ-dimensional standard normal probability distribution function. Therefore, ρ is the Pearson correlation between two normal scores $\Phi_{[1]}^{-1}(\varphi)$ measures the dependence between $y_1^\star$ and $y_2^\star$ through the monotone nonlinear transformation (5), *i.e.*, $\Phi_{[1]}^{-1}\left[F_i\left(y_i + (u-1)\right)\right]$. Thus, from Sklar's result (7), the Gaussian copula (9), and equations (5)–(6), it is possible to write the joint probability density function of the continued variables as the product of the marginal density functions and the copula density:

$$f^\star(y_1^\star, y_2^\star) = f_1^\star(y_1^\star) f_2^\star(y_2^\star) \frac{\partial^2 C^\star\left(F_1^\star(y_1^\star), F_2^\star(y_2^\star)\right)}{\partial F_1^\star(y_1^\star) \partial F_2^\star(y_2^\star)} = f_1(y_1) f_2(y_2) \phi_{[2]}\left(\Phi_{[1]}^{-1}(\varphi_1), \Phi_{[1]}^{-1}(\varphi_2); \rho\right) , \tag{10}$$

where $\phi_{[2]}(\cdot)$ is the bivariate standard normal probability density function. This expression simplifies the log-likelihood function needed to estimate this bivariate count data model with double Poisson marginals because it becomes additively separable in the different parameters in such a way that it allows to estimate them sequentially. From equation (10), the contribution of each observation j to the log-likelihood is:

$$\sum_{i=1}^{2} \log\left[f_i(y_{ij}, \mu_{ij}, \theta_i | \boldsymbol{x_i})\right] + \log\left[\phi_{[2]}\left(\Phi_{[1]}^{-1}\left[F_1\left(y_{1j} + (u_j - 1)\right)\right], \Phi_{[1]}^{-1}\left[F_2\left(y_{2j} + (u_j - 1)\right)\right]; \rho\right)\right] , \tag{11}$$

where $f_i(\cdot)$ is the univariate probability density function of the double Poisson given by equation (1a). Hence, the log-likelihood function of this model has two differentiated components consisting of the log-likelihood functions of two separate double Poissons plus the term involving the copula that exclusively determines ρ. Iterative, reweighted ordinary least squares provides with a fast and consistent estimate of $\hat{\boldsymbol{\beta}}_{\mathbf{1}}$ and $\hat{\boldsymbol{\beta}}_{\mathbf{1}}$ using the exponential mean function (1b). Substituting these estimates into (3) produces consistent estimates of $\hat{\theta}_1$ and $\hat{\theta}_2$. The estimates $\hat{\varphi}_{1j}$ and $\hat{\varphi}_{2j}$ are simply computed according to the probability integral transformation (4) after adding the uniformly distributed random draw u_j to each count variable. Then ρ

is consistently estimated in a last stage simply as Pearson's linear correlation between the corresponding univariate normal scores:[7]

$$\hat{\rho} = \mathrm{CORR}\left(\Phi^{-1}_{[1]}(\hat{\varphi}_1), \Phi^{-1}_{[1]}(\hat{\varphi}_2)\right) . \quad (12)$$

Therefore, the combination of a Gaussian copula with double Poisson marginals reduces the estimation of a multivariate count data regression model to an easy sequence of univariate generalized linear regressions and the robust estimation of the correlation between the error terms of these equations. This procedure is easy to implement and produces consistent estimates of the parameters in three stages that allow accommodating any combination of over and underdispersion of counts plus any correlation pattern between the counts.

3.1 Alternative Approaches

Before ending this section it might be useful to discuss how this model compares to potential alternative approaches. The proposed model has several advantages over the existing alternatives:

1. Simultaneous ordinary least square estimation. This approach ignores the countable nature of the endogenous variables; it allows for negative outcomes; the implied probability of any particular outcome is zero; it fails to address the heteroskedasticity inherent to count data; and if the conditional mean function is indeed log-linear, it provides with inconsistent estimates. See Cameron and Trivedi (1998, §3.7.1) and Winkelmann (2008, §3.1.2).

2. Bivariate ordered probit model. The basic problem with this approach is that it imposes an underlying continuous data generating process to events that are clearly determined by a point process. Estimates ignore the cardinality of the count process and thus wrongly assume that the sequence "1, 4, 37" is as likely as "0, 1, 2"; it is not possible to predict the probability of the counts that are not included in the sample; and the model is not identified if all equations include the same regressors.See Cameron and Trivedi (1998, §3.6.2) and Winkelmann (2008, §3.1.2).

3. Some of the existing multivariate models for count data variables. The few existing ones only address the possibility of overdispersion. In addition, the determinants of overdispersion are the same behind the correlation estimate and thus the latter is necessarily positive. See Cameron and Trivedi (1998, §8.3).

4. Non-Gaussian copula. Miravete (2008) uses the multivariate Sarmanov distribution to identify the correlation among counts independently of over or underdispersion. However, the estimation is

[7] The "continuation" (4) is a monotone increasing transformation that preserves the concordance of pairs $\{y_i, y_{i'}\}$ and their counterparts $\{y_i^\star, y_{i'}^\star\}$. Thus, Pearson's linear correlation $\rho(y_i, y_{i'})$ will generally differ from $\rho(y_i^\star, y_{i'}^\star)$ since (4) is not exactly a linear transformation. For this reason I also report Kendall's τ in Section 4 since this nonlinear association measure is invariant to any monotone transformation.

rather involved as it requires the use of constrained maximum likelihood because while correlation is still allowed to take any sign, the estimates of the regressors limit the range of variation of the correlation for the copula to be properly defined.

4 Estimation Results

Table 4 reports the estimates of this model for the actual number of tariff options and Table 5 does the same for the effective number of tariff options. In both cases, five specifications are estimated, starting from the simplest one that only controls for the effect of consumer heterogeneity through the equilibrium measure associated to the curvature of the tariff and market penetration. Firm specific characteristics are then added (FIRM-AGE, BELL), inertia from previous pricing practices by the incumbent when it enjoyed monopoly power (LASTMONO or LSTEFMON), time dummies (YEAR88, YEAR92), and city fixed effects (not reported).

Results are generally robust across specifications regardless of whether we focus on total or effective tariff options. First, there is evidence consistent with the existence of asymmetry of information driving the pricing decision although it manifests differently depending on the type of tariffs we consider. Estimates indicate that the distribution of actual tariff plans is underdispersed ($\phi_i > 1$ for both firms) while the distribution of effective tariff options is overdispersed ($\phi_i < 1$ also for both firms). These results are consistent across specifications survive controlling for unobserved heterogeneity through market and time fixed effects.

Some other results do not survive controlling for the existence of unobserved heterogeneity. For instance, there is a significant effect of learning attributable to the experience accumulated by each firm in a particular market only if we ignore time effects, which means that the few instances when the FIRM-AGE variable is significant, it is only capturing the effect of an omitted time trend. Once we control for time fixed effects, accumulated experience does not help explaining pricing decision regarding either total or just effective tariff plans.

Some variables have non-significant effects consistently across specifications in both tables. Thus, the number of tariff options offered to consumers is independent of the ownership of the carrier. Former BELL carriers do not present a significantly different pricing behavior than independently owned carriers This result is consistent with the view that firms face common commercialization costs that influence their decision about how many tariff plans to offer independently of the ownership of the firms (and other controls).[8] The other consistently non-significant results indicate that in this early industry differences of participation (COVERAGE) across markets and time have no influence on the offering of tariffs. Similarly,

[8] Certainly this is only one possible explanation. An alternative one is that all these firms, even the independent carriers, have managers with common experiences in the recently divested AT&T that may lead them to take similar pricing decisions.

Table 4: Number of Actual Tariffs

	I		*II*		*III*		*IV*		*V*	
Variables	Inc.	Ent.	Inc.	Ent.	Inc.	Ent.	Inc.	Ent.	Inc.	Ent.
CONSTANT	2.8732	2.1168	2.0417	1.5238	2.0502	1.4288	2.0672	1.3016	1.4071	0.2398
	(3.97)	(2.76)	(2.67)	(1.96)	(2.68)	(1.83)	(2.67)	(1.61)	(0.95)	(0.14)
FIRM-AGE			0.0155	0.0195	0.0156	0.0199	–0.0041	0.0024	–0.0169	–0.0300
			(3.08)	(3.91)	(3.10)	(3.97)	(0.20)	(0.14)	(0.52)	(1.05)
BELL			0.3392	0.2793	0.3997	0.3208	0.5539	0.4046	0.2036	0.0654
			(0.98)	(0.85)	(1.06)	(0.97)	(1.36)	(1.18)	(0.27)	(0.08)
LASTMONO					–0.0352	0.0733	0.0035	0.0564	0.1287	–0.0622
					(0.39)	(0.90)	(0.04)	(0.68)	(0.51)	(0.28)
YEAR88							0.5862	0.3447	1.0148	1.1535
							(0.85)	(0.57)	(0.98)	(1.34)
YEAR92							1.5201	1.2786	2.6442	3.5232
							(1.02)	(0.99)	(1.08)	(1.70)
AP_{peak}	0.8543	0.6592	1.2663	0.8283	1.2272	0.8507	1.2290	0.8507	2.4152	0.8221
	(1.50)	(1.28)	(2.15)	(1.58)	(2.05)	(1.61)	(2.06)	(1.57)	(3.01)	(1.15)
$\text{AP}_{\text{off-peak}}$	–0.0071	–0.0196	–0.0112	–0.0280	–0.0112	–0.0296	–0.0089	–0.0279	–0.0107	–0.0280
	(0.39)	(2.07)	(0.70)	(2.93)	(0.70)	(3.04)	(0.54)	(1.57)	(0.66)	(2.60)
COVERAGE	0.4208	–0.2499	0.7232	0.1144	0.7144	0.1725	0.7139	–0.0148	0.5537	–0.7737
	(0.63)	(0.36)	(1.07)	(0.16)	(1.05)	(0.24)	(1.04)	(0.02)	(0.39)	(0.49)
ϕ_i	1.1405	1.1392	1.1080	1.1052	1.1079	1.1017	1.1032	1.0969	1.0586	1.0619
	(12.78)	(13.03)	(11.99)	(12.15)	(11.98)	(12.06)	(11.87)	(11.95)	(10.84)	(11.11)
ρ, *Kendall's* τ	0.5573	0.3861	0.3235	0.2381	0.2763	0.2141	0.3646	0.2303	0.2885	0.1948
	(6.16)	(5.66)	(3.29)	(3.49)	(2.78)	(3.14)	(3.74)	(3.38)	(2.63)	(2.86)
$-\ln L$	98.7482		96.8512		95.8367		96.3664		90.4110	
Market Dummies	*No*		*No*		*No*		*No*		*Yes*	
LM	0.3926	1.7660	2.8230	1.8348	1.2576	2.0586	0.6362	1.0866	5.0013	2.1179
$[p - value]$	[0.822]	[0.414]	[0.244]	[0.400]	[0.533]	[0.357]	[0.728]	[0.581]	[0.082]	[0.347]
KS	0.7263	0.8494	1.1550	1.2201	1.1474	1.2956	1.2036	1.2541	1.4786	1.3922
$[p - value]$	[0.667]	[0.466]	[0.139]	[0.102]	[0.147]	[0.070]	[0.110]	[0.086]	[0.025]	[0.041]

Marginal effects evaluated at the sample mean of regressors. Endogenous variables are the actual and effective number of tariff options, respectively minus one (to include zeroes). Absolute value, t-statistics are reported between parentheses. *LM* is the regression-based, heteroskedastic-robust, Lagrange multiplier test of endogeneity of Wooldridge (1997) for AP_{peak}, $\text{AP}_{\text{off-peak}}$, and COVERAGE. *LM* is asymptotically distributed as a χ^2_3 distribution under the null hypothesis of exogeneity. *KS* is the Kolmogorov-Smirnov test of uniform distribution of the probability integral transformations φ_i. In both cases, p-values are shown between brackets. Sample size includes 99 duopoly markets.

the prior pricing policy of the incumbent while it enjoyed a monopoly position (LASTMONO or LSTEFMON) has no effect on pricing during the duopoly phase of the market. Strategic considerations may dominate any other effect and thus, the incumbent does not generally follow the same pricing policy than before and the entrant does not find the previous pricing of the incumbent informative about the nature of consumers' valuation of the service.

The working assumption in industries like cellular telephony is that there is substantial asymmetry of information between consumers and the service provider. Unfortunately no individual data on tariff choice or consumption are available and thus I need to resort to an equilibrium measure, the curvature of the tariff, to control for the existence of this important unobservable effect. It is therefore and important result to find that there is a significant effect of AP_{peak} on the total and effective number of tariff options. This result holds across specifications and is robust even after controlling for market fixed effects. This is

Table 5: Number of Effective Tariffs

Variables	*I* Inc.	*I* Ent.	*II* Inc.	*II* Ent.	*III* Inc.	*III* Ent.	*IV* Inc.	*IV* Ent.	*V* Inc.	*V* Ent.
CONSTANT	1.4622	1.2785	0.8087	0.6788	0.8175	0.5361	0.8606	0.4415	0.1470	–0.1433
	(2.05)	(1.75)	(1.07)	(0.91)	(1.08)	(0.72)	(1.13)	(0.57)	(0.10)	(0.08)
FIRM-AGE			0.0172	0.0193	0.0173	0.0199	0.0066	0.0059	–0.0252	–0.0184
			(3.62)	(4.07)	(3.64)	(4.16)	(0.35)	(0.34)	(0.81)	(0.64)
BELL			–0.1584	0.2540	–0.0854	0.3160	0.0033	0.3844	–0.3179	–0.8135
			(0.49)	(0.81)	(0.24)	(1.00)	(0.01)	(1.17)	(0.42)	(0.96)
LSTEFMON					–0.0431	0.1092	–0.0229	0.0947	0.2067	–0.0499
					(0.50)	(1.43)	(0.24)	(1.21)	(0.85)	(0.22)
YEAR88							0.1424	0.2254	1.0864	0.8265
							(0.22)	(0.38)	(1.09)	(0.96)
YEAR92							0.7851	1.0067	3.2462	2.6448
							(0.55)	(0.80)	(1.40)	(1.27)
AP_{peak}	1.1314	0.7672	1.6295	0.9437	1.5801	0.9782	1.5899	0.9905	2.5052	1.0748
	(2.18)	(1.61)	(3.03)	(1.93)	(2.89)	(1.97)	(2.90)	(1.94)	(3.50)	(1.60)
$\text{AP}_{\text{off-peak}}$	–0.0051	–0.0019	–0.0059	–0.0111	–0.0059	–0.0135	–0.0064	–0.0122	–0.0089	–0.0096
	(0.35)	(0.16)	(0.38)	(0.94)	(0.38)	(1.13)	(0.40)	(1.01)	(0.59)	(0.77)
COVERAGE	0.0193	–0.2713	0.2738	0.0872	0.2588	0.1800	0.2189	0.0152	0.0082	–0.5580
	(0.03)	(0.40)	(0.40)	(0.13)	(0.37)	(0.26)	(0.31)	(0.02)	(0.01)	(0.34)
ϕ_i	1.0126	1.0252	0.9655	0.9631	0.9638	0.9558	0.9577	0.9495	0.9103	0.9086
	(12.62)	(12.99)	(11.29)	(11.22)	(11.24)	(11.04)	(11.06)	(10.87)	(9.78)	(9.72)
ρ, *Kendall's* τ	0.4164	0.3024	0.1817	0.1540	0.2058	0.1866	0.2701	0.1878	0.2623	0.0835
	(4.34)	(4.43)	(1.80)	(2.26)	(2.05)	(2.74)	(2.71)	(2.75)	(2.91)	(1.22)
$-\ln L$	107.2181		99.2699		97.6680		99.0183		92.2950	
Market Dummies	*No*		*No*		*No*		*No*		*Yes*	
LM	2.2560	3.6916	1.2761	0.2275	0.3597	2.5924	2.9800	1.5333	2.9316	1.8541
$[p - value]$	[0.324]	[0.158]	[0.528]	[0.989]	[0.835]	[0.274]	[0.225]	[0.465]	[0.231]	[0.396]
KS	0.6425	0.9133	1.0605	1.4107	1.1680	1.4051	1.0824	1.5930	1.7307	1.9089
$[p - value]$	[0.804]	[0.375]	[0.211]	[0.037]	[0.131]	[0.039]	[0.192]	[0.013]	[0.005]	[0.001]

Marginal effects evaluated at the sample mean of regressors. Endogenous variables are the actual and effective number of tariff options, respectively minus one (to include zeroes). Absolute value, t-statistics are reported between parentheses. *LM* is the regression-based, heteroskedastic-robust, Lagrange multiplier test of endogeneity of Wooldridge (1997) for AP_{peak}, $\text{AP}_{\text{off-peak}}$, and COVERAGE. *LM* is asymptotically distributed as a χ^2_3 distribution under the null hypothesis of exogeneity. *KS* is the Kolmogorov-Smirnov test of uniform distribution of the probability integral transformations φ_i. In both cases, p-values are shown between brackets. Sample size includes 99 duopoly markets.

important because the peak time band comprises at this early stage of the industry more than twelve hours a day and constitutes the bulk of the market, clearly made mostly of business and high income individuals. It indicates that the more asymmetric is the information regarding tastes of the clientele base (the more disperse the distribution of consumer types is), the more tariff options are needed to successfully segment the relevant groups and extract most of the informational rents from individuals with different willingness to pay for cellular service. This effect is common to both firms although always stronger for the incumbent.

Results involving AP_{peak}, $\text{AP}_{\text{off-peak}}$, and COVERAGE could however be questioned on the basis that these variables may not be exogenous as the curvature measure depends on the tariff options actually offered and the decision to participate in the market is probably a function of the menu of tariff options available to different customers. Fortunately, the Lagrange multiplier test of Wooldridge (1997) concludes that these three variables can always be considered exogenous regressors. Thus, pricing has to be understood as the optimal strategy to discriminate consumers with a given distribution of tastes.[9]

Fixed time and market fixed effects complete our attempt to control for the existence of observable heterogeneity. Specification V controls for the existence of unobserved heterogeneity by means of a fixed effect count data regression model.[10] Thus, the significant positive correlation between pricing strategies of incumbents and entrants is not the result of correlations among unobserved returns associated to these pricing strategies, but rather it is a strong evidence that favor the interpretation that the number of tariff plans, whether total or just the effective ones, are considered strategic complements by the competing firms. Positive correlation is roughly about 0.27 on average (regardless of which type of tariffs we consider) and always strongly significant. This result implies that in addition to addressing the significant asymmetry of information, firms always have an incentive to offer a similar number of tariff options than the competitor. Beyond asymmetry of information considerations firms imitate each other when it comes to the implementation of the optimal pricing mechanism in a competitive environment. Thus, if we abstract from any other tariff feature but the number of options available to consumers, the evidence favors the *minimal tariff differentiation* result of Armstrong and Vickers (2001) and Rochet and Stole (2002) at this early stage of development of the cellular telephone industry. Results do not support the alternative view of Yang and Ye (2008), who argue that firms may differentiate from each other through the design of the pricing options and thus grow at the expense of competitors through a business stealing effect. But Yang and Ye (2008) also argue that this effect will most likely happen in mature markets. In this early industry the market is far from saturated and just two firms can still find a substantial share of potential customers to sign on without the need of having to steal customers from the competitor through other pricing tactics. In support of this view, notice that the effect of COVERAGE is never significant.

Table 4 and Table 5 also include non-parametric estimates of Kendall's τ to measure the nonlinear association between the Pearson's residuals. Its value is generally smaller than those of the linear correlation estimate, ρ. The advantage of Kendall's τ over ρ is that it is invariant to any monotone transformation such as the continuation equation (4). Thus, whether we use the original counts y_i's or the continued $y_i^{\star}$'s, the estimated τ's are identical.

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[9] To construct this test I need to instrument these three variables with market specific demand and cost shifters such as income, size of the market, or administrative costs that may be related to the heterogeneity of consumers and cost of offering the plans but that are orthogonal to the number of plans offered. If demand shocks are market specific, as opposed to nationally driven, the characteristics of the tariffs of the competitors in other markets during past periods can also be used as valid instruments as suggested by Hausman, Leonard, and Zona (1994) and Hausman (1996). I thus also include as instrument the average of the curvature of tariffs across markets where the competing firm is present.

[10] Contrary to other nonlinear models, maximum likelihood estimates of a Poisson regression with multiplicative fixed effects are still unbiased and consistent even in the presence of incidental parameters. See Cameron and Trivedi (1998, §9.3.1) for a formal proof of this result. However, the current specification does not suffer from this problem as I do not include a full set of firm-specific effects.

Finally, I tried to test whether the proposed specification of the model could be rejected. If the model is properly specified, the probability integral transformation defined by equation (5) should be uniformly distributed. *KS* is the Kolmogorov-Smirnov test of uniform distribution of the probability integral transformations φ_i. The test is clearly rejected when we include fixed effects, which may question the results of specification *V*. In all other cases, the Kolmogorov-Smirnov test is mostly not rejected and never rejects the uniformity of the probability integral transformations of the distribution of number of tariffs of the incumbent and entrant simultaneously. I thus conclude that the bivariate double Poisson model with Gaussian copula is generally well specified.

5 Concluding Remarks

This paper has described the pricing tactics of competing firms in order to determine whether they can be considered strategic complements or substitutes. Results support the idea of strategic complementarity regardless of whether we consider the total or just the effective tariff options offered. Thus, at least for an early industry, firms do not differentiate from each other through the design of tariffs and they tend to offer a similar number of tariffs. Whether this similarity in pricing practices arises because of an attempt to collude the market falls beyond what I can analyze with this data. The minimal differentiation in pricing does not mean that tariffs necessarily have to be numerous and/or complicated. Tariffs can also be equally simple as the populated diagonals of Table 1 shows. The econometric analysis conducted in this paper concludes that the positive correlation among tariff plans of competing firms is robust to the existence of observable and unobservable heterogeneity and that the strategic complementarity between pricing strategies is not the result of some market specific effects but rather the result of strategic incentives between the competing cellular carriers.

The paper has also introduced an effective and easily implementable estimation method to obtain consistent estimates in multivariate count data settings, which is of particular interest for applied econometricians dealing with the existence of complementarity of strategies since they are very frequently non-continuous in nature. This approach solves a long standing problem in the multivariate count data regression literature and allows for correlations among counts of any sign. In addition it can also accommodate the effect of unobservable heterogeneity leading not only to the common overdispersion of counts, but also to underdispersion.

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