Testing for Complementarity when Strategies are Dichotomous[⋆]

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Submitted: November 25, 2008 — *Accepted:* September 2, 2009

Abstract

We show that it is not possible to extend Arora's (1996) reduced form test for the existence of complementarity to evaluate the relationship between a couple of dichotomous strategies as it leads to an incoherent simultaneous discrete response model.

Keywords: Dichotomous Strategies, Complementarity, Incoherent Models.

JEL Codes: C35, O31, O32.

[⋆] José C. Pernías gratefully acknowledges financial support from the Spanish Ministerio de Ciencia y Tecnología (ECO2008.06057/ECON) and from Fundació Caixa Castelló (P1 1A 2007-19).

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1 Introduction

In an influential paper, Arora and Gambardella (1990) suggest a test for complementarity that has since been routinely computed. This test concludes that two strategies are complementary if they are correlated conditional on the observable firm or market features that potentially influences their adoption and/or use. Therefore, the test evaluates whether the residuals of regressions of endogenous strategies on firms' observable characteristics are positively correlated. Arora (1996) argued in favor of the robustness of this reduced form test because it does not impose restrictions on the profit function beyond concavity and also because it is straightforward to generalize to evaluate the case where more than two decision variables are involved.

There is, however, an important underlying assumption in the analysis of Arora (1996) that has been overlooked so far: strategy choices are implicitly assumed to be continuous variables.[1] Milgrom and Roberts (1990) point out that many of the decisions that firms make, such as innovations, are discrete in nature. The obvious extension of the reduced form approach of Arora and Gambardella (1990) is to estimate a simultaneous discrete response model, such as a bivariate probit. Thus, for instance, Cassiman and Veugelers (2006, §5.3.1) estimate a bivariate probit model where firms decide whether they engage in in-house R&D or acquire external knowledge though licensing. To conduct this test, they implicitly assume that there are a couple of underlying continuous variables that determine whether firms engage in one of the two discrete choices, both, or neither of them.[2]

Unfortunately, extending the reduced form test of complementarity to measuring association of dichotomous variables leads to incoherence problems. Intuitively, incoherence arises when it is not possible to associate any realization of the vector of error terms with a unique combination of strategies, so that the sum of probabilities associated with all possible combinations of strategy choices adds up to more than one. In this paper we show that in order to avoid such incoherence problems, the underlying equilibrium model of profit maximization should exclude any complementarity relationship among strategies so that the estimated correlation parameter in a bivariate probit regression is only evidence of correlation among the unobserved returns of each strategy but not of complementarity.

[1] Indeed, the endogenous variables of Arora and Gambardella (1990) are not continuous but countable. Miravete (2008) shows how to test for correlation across strategies of different firms when they have a countable nature.

[2] Other examples of this approach are Åstebro, Colombo, and Seri (2005), Falk (2006), Grimpe and Hussinger (2008), and Zúñiga, Guzmán, and Brown (2007).

2 Econometric Model

Arora (1996) notes that the reduced form test of complementarity is a sufficient test only when two strategies are involved. The reason is that if all correlations are not positive, direct and indirect effects may offset each other. To avoid any ambiguity we exclusively focus on the bivariate case.

2.1 The Profit Function

In order to maximize profits firm i decides whether to use any combination of two dichotomous strategies. Let's denote these strategies $\{x_{1\,i}, x_{2\,i}\}$. These strategies could represent the adoption of product and process innovations, embarking in an R&D program and acquiring technology through licensing, or developing new product qualities and implementing a new managerial incentive program. If firm i chooses to adopt the first (second) strategy, $x_{1\,i} = 1$ ($x_{2\,i} = 1$). Therefore, firm i chooses one out of four combination of strategies: (i) adoption of the first strategy only, $\{1, 0\}$; (ii) adoption of the second strategy only, $\{0, 1\}$; (iii) adoption of both strategies, $\{1, 1\}$; and (iv) adoption of neither of them, $\{0, 0\}$. The profit function of firm i is

$$\pi_i(x_{1\,i}, x_{2\,i}) = (\theta_1 + \epsilon_{1\,i})x_{1\,i} + (\theta_2 + \epsilon_{2\,i})x_{2\,i} + \delta_{12}x_{1\,i}x_{2\,i}. \tag{1}$$

This is a general approximation to any profit function in the spirit of the "non-functional form approach" vindicated by Vives (2008). If the firm adopts strategy x_i, $i = 1, 2$, it obtains a total return of $(\theta_i + \epsilon_i)$, where θ_i is the return that can be explained by observable characteristics of the firm or market where it operates, and ϵ_i is a structural error that represents the returns that are observed by firms but not by econometricians. These unobservable return components explain why firms with identical observable characteristics (θ_1, θ_2) may end up choosing different combinations of strategies $\{x_{1\,i}, x_{2\,i}\}$ and reaching different profit levels, π_i. The profit function also includes an interaction between these dichotomous strategies, the parameter δ_{12}, whose sign determines whether the profit function is supermodular or submodular in $\{x_{1\,i}, x_{2\,i}\}$. Thus, $x_{1\,i}$ and $x_{2\,i}$ are complements if $\delta_{12} > 0$, *i.e.*, the return of adopting $x_{1\,i}$ is higher if the firms also adopts $x_{2\,i}$. Alternatively, $x_{1\,i}$ and $x_{2\,i}$ are substitutes if $\delta_{12} < 0$.

The estimation of this structural model requires that we associate each combination of errors $(\epsilon_{1\,i}, \epsilon_{2\,i})$ to one and only one combination of strategies $\{x_{1\,i}, x_{2\,i}\}$. Suppose that a firm adopts both strategies. It is then necessary that the following conditions hold:

$$\pi(1, 1) > \pi(1, 0), \tag{2a}$$

$$\pi(1,1) > \pi(0,1), \tag{2b}$$

$$\pi(1,1) > \pi(0,0). \tag{2c}$$

Making use of (1), these conditions correspond to the following realizations of the unobserved returns of each strategy $(\epsilon_{1\,i}, \epsilon_{2\,i})$:

$$\epsilon_{1\,i} > -\theta_1 - \delta, \tag{3a}$$

$$\epsilon_{2\,i} > -\theta_2 - \delta, \tag{3b}$$

$$\epsilon_{1\,i} + \epsilon_{2\,i} > -\theta_1 - \theta_2 - \delta_{12}. \tag{3c}$$

We can repeat this analysis for any of the other three possible innovation profiles. This structural approach takes into account the discrete nature of the innovation decisions and thus, it incorporates the inequalities (2a)–(2c) in defining the innovation profile of firms as the result of the relative profitability of adopting each combination of innovation strategies.[3] The advantage of this structural approach is that it differentiates between δ_{12}, the complementarity between $x_{1\,i}$ and $x_{2\,i}$, and ρ_{12} the correlation between unobservable returns $\epsilon_{1\,i}$ and $\epsilon_{2\,i}$.

2.2 A Couple of Continuous Latent Endogenous Variables

Alternatively, many empirical studies model the effect of observable (and perhaps unobservable) firm and market characteristics on some underlying continuous profitability of adopting an innovation. Firms adopt the innovation only if such profitability exceeds some threshold, *i.e.*, if profits of adopting are positive. Let $x^\star_{1\,i}$ denote the unobservable increase in profits related to the adoption of strategy 1 ($x^\star_{2\,i}$ is defined similarly):

$$x^\star_{1\,i} = \pi(1, x_{2\,i}) - \pi(0, x_{2\,i}). \tag{4}$$

From the profit function (1), we get:

$$x^\star_{1\,i} = \theta_1 + \delta_{12} x_{2\,i} + \epsilon_{1\,i}, \tag{5}$$

Next, we define the adoption indicators as a function of whether firms obtain positive profits if they engage in each innovation strategy:

[3] Notice that the third condition (3c) is non-binding when $\delta_{12} \leq 0$. Miravete and Pernías (2006) estimate a similar model assuming that $(\epsilon_{1\,i}, \epsilon_{2\,i})$ are jointly distributed according to a multivariate normal distribution with unrestricted correlation.

$$x_{j\,i} = \begin{cases} 1 & \text{if} \quad x^{\star}_{j\,i} > 0, \\ 0 & \text{if} \quad x^{\star}_{j\,i} \le 0, \end{cases} \qquad (j = 1, 2). \tag{6}$$

For the empirical analysis, the directly observable components of the returns, θ_1 and θ_2 can be made a linear function of observable firm and market characteristics, $\boldsymbol{z}_i$. These relations can be expressed in matrix form as:

$$\boldsymbol{\theta}_i = \boldsymbol{\Theta} \boldsymbol{z}_i, \qquad (i = 1, 2). \tag{7}$$

where $\boldsymbol{\theta}_i = (\theta_1, \theta_2)'$, and where $\boldsymbol{\Theta}$ is a the matrix of exclusion restrictions of regressors on each direct observable return equation. Thus, stacking the optimal adoption rules, (4), can be written in matrix form as:

$$\boldsymbol{x}^{\star}_i = \boldsymbol{\Theta} \boldsymbol{z}_i + \boldsymbol{\Gamma} \boldsymbol{x}_i + \boldsymbol{e}_i, \tag{8}$$

where $\boldsymbol{x}^{\star}_i = (x^{\star}_{1\,i}, x^{\star}_{2\,i})'$, and:

$$\boldsymbol{\Gamma} = \begin{pmatrix} 0 & \delta_{12} \\ \delta_{12} & 0 \end{pmatrix}. \tag{9}$$

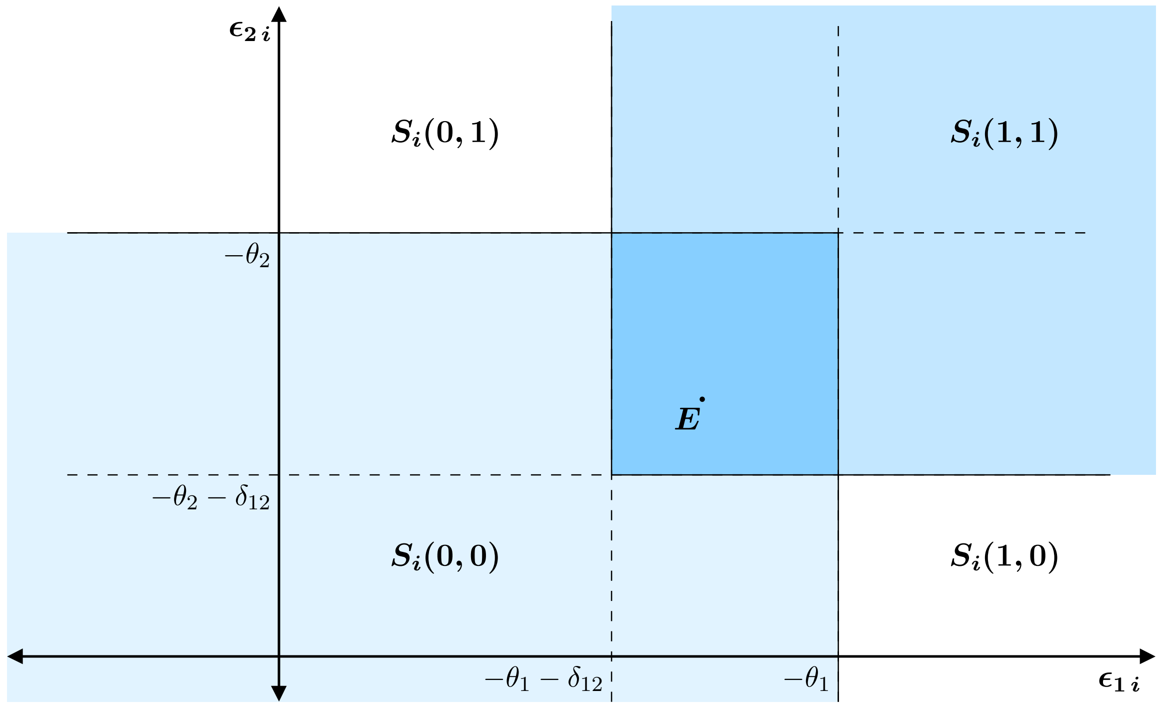
Equation (8) together with the observation rules (6), define a model of simultaneous equations where both endogenous variables are only partially observable. Notice that the right hand side of (8) only includes the observable indicators, $x_{1\,i}$ and $x_{2\,i}$:

$$\boldsymbol{x}_i = \left(\boldsymbol{I}(x^{\star}_{1\,i} \ge 0), \boldsymbol{I}(x^{\star}_{2\,i} \ge 0)\right)'. \tag{10}$$

2.3 Meaning of Incoherence

Schmidt (1981, §8.2) discusses specific restrictions that the parameters of this class of models need to fulfill. In particular, some minimum degree of recursion is required so that for any vector $\boldsymbol{z}_i$, the realization of errors $\boldsymbol{e}_i$ uniquely determine the choice of endogenous variables. The coherence of the model requires that the principal minors of matrix $\boldsymbol{\Gamma}$ are equal to zero. It is straightforward to show that the principal minors of $\boldsymbol{\Gamma}$ can only be zero if parameter δ_{12} is zero, *i.e.*, the case where the profit function is linear in the strategies and no complementarities between the dichotomous strategies exist. Therefore, using the approach summarized by equations (6) and (8) where we model the latent and continuous willingness to adopt dichotomous strategies, it is not possible to address whether they are complements; it is *only* possible to address the existence of potential correlations between these strategies due to the unobservable return components.

Figure 1: Incoherence of a System of Equations



We can illustrate the meaning of the coherence problem by substituting the definitions of $x^{\star}_{1\,i}$ (and similarly for $x^{\star}_{2\,i}$) given by (5) into the indicator function (6), so that:

$$x_{1\,i} = \begin{cases} 1 & \text{if} \quad \epsilon_{1\,i} > -\theta_1 - \delta_{12} x_{2\,i}, \\ 0 & \text{if} \quad \epsilon_{1\,i} \leq -\theta_1 - \delta_{12} x_{2\,i}. \end{cases} \tag{11}$$

Next, define

$$S_i(1,1) = \big\{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \arg\max \pi_i(x_{1\,i}, x_{2\,i}) = (1,1)\big\}, \tag{12}$$

as the set of values of $\epsilon_{1\,i}$ and $\epsilon_{2\,i}$ that induce firm i to adopt both strategies simultaneously. The combinations of $\epsilon_{1\,i}$ and $\epsilon_{2\,i}$ leading to the remaining innovation profiles, $S_i(1,0)$, $S_i(0,1)$, and $S_i(0,0)$ can be defined in a similar manner to equation (12). Making use of (11) we obtain:

$$S_i(1,1) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} > -\theta_1 - \delta_{12},\ \epsilon_{2\,i} > -\theta_2 - \delta_{12}\}, \tag{13a}$$

$$S_i(1,0) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} > -\theta_1,\ \epsilon_{2\,i} < -\theta_2 - \delta_{12}\}, \tag{13b}$$

$$S_i(0,1) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} < -\theta_1 - \delta_{12},\ \epsilon_{2\,i} > -\theta_2\}, \tag{13c}$$

$$S_i(0,0) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} < -\theta_1,\ \epsilon_{2\,i} < -\theta_2\}. \tag{13d}$$

These four regions in the $(\epsilon_{1\,i}, \epsilon_{2\,i})$ space are represented in Figure 1 for the case where parameter δ_{12} is positive. This figure shows that subsets $S_i(1,1)$ and $S_i(0,0)$ overlap each other. This means that the combinations of $\epsilon_{1\,i}$ and $\epsilon_{2\,i}$ leading to the optimal choice of both strategies and the optimal choice of not adopting any of them, respectively are not disjoint sets and thus, identical realizations of $\epsilon_{1\,i}$ and $\epsilon_{2\,i}$ such as that represented by point $\boldsymbol{E}$, may lead to the optimal choice of completely different innovation profiles.[4] This is the exact meaning of incoherence of the system defined by (8) and (6). Heckman (1978) shows that a necessary and sufficient condition for this bivariate discrete choice model to be "properly" defined is:

$$\mathrm{Prob}[S_i(1,1)] + \mathrm{Prob}[S_i(1,0)] + \mathrm{Prob}[S_i(0,1)] + \mathrm{Prob}[S_i(0,0)] = 1. \tag{14}$$

To conclude we address the special case where complementarities are absent, *i.e.*, when $\delta_{12} = 0$. In this case matrix $\boldsymbol{\Gamma}$ is a null matrix and equation (8) becomes:

$$\boldsymbol{x}_i^* = \boldsymbol{\Theta}\boldsymbol{z}_i + \boldsymbol{e}_i. \tag{15}$$

Without complementarities, we have a seemingly unrelated equations model rather than a simultaneous discrete choice model. The optimal choices of strategies are independent of each other beyond any potential correlation that may exist among the unobservable returns to each strategy. As before, the rules defining our innovation indicators simplify to:

$$x_{j\,i} = \begin{cases} 1 & \text{if} \quad \epsilon_{j\,i} > -\theta_j, \\ 0 & \text{if} \quad \epsilon_{j\,i} \leq -\theta_j. \end{cases} \qquad (j = 1, 2). \tag{16}$$

Values of $\epsilon_{1\,i}$ and $\epsilon_{2\,i}$ leading to each of the four possible innovation profiles are now:

$$S_i(1,1) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} > -\theta_1,\ \epsilon_{2\,i} > -\theta_2\}, \tag{17a}$$

$$S_i(1,0) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} > -\theta_1,\ \epsilon_{2\,i} < -\theta_2\}, \tag{17b}$$

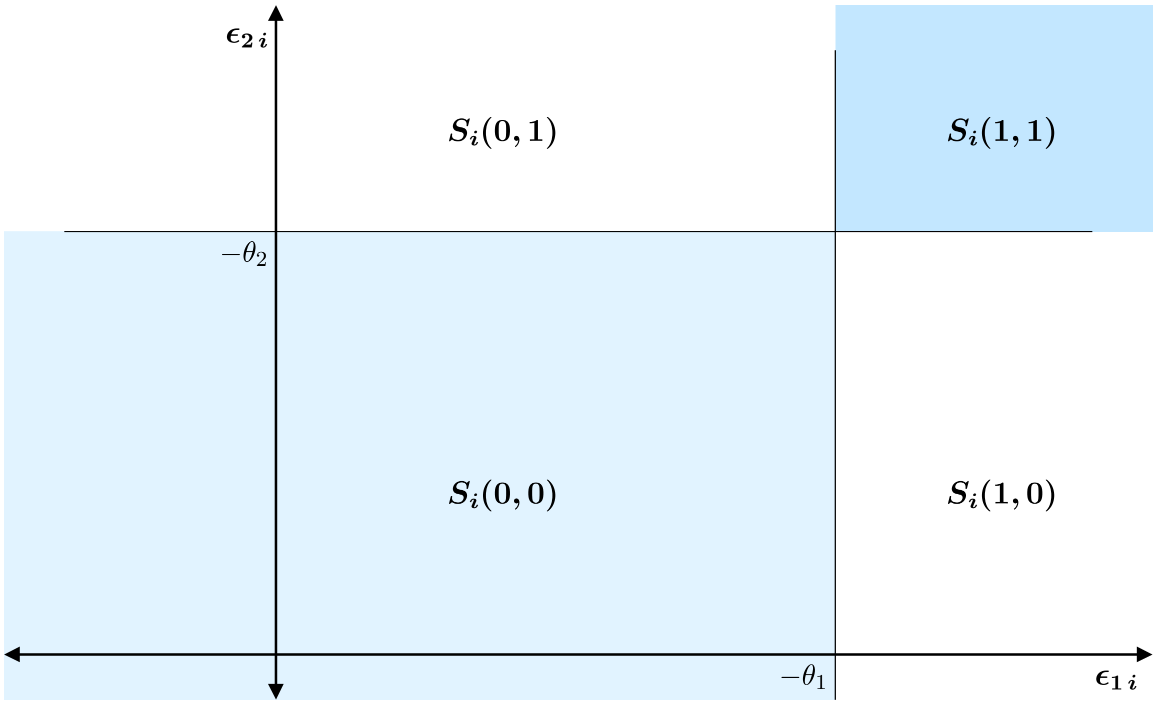
$$S_i(0,1) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} < -\theta_1,\ \epsilon_{2\,i} > -\theta_2\}, \tag{17c}$$

$$S_i(0,0) = \{(\epsilon_{1\,i}, \epsilon_{2\,i}) : \epsilon_{1\,i} < -\theta_1,\ \epsilon_{2\,i} < -\theta_2\}. \tag{17d}$$

Figure 2 shows that without complementarity between $x_{1\,i}$ and $x_{2\,i}$, any combination of values of $\epsilon_{1\,i}$ and $\epsilon_{2\,i}$ uniquely determines a single strategy choice and that regions (17a)–(17d) are disjoint and divide the whole $\epsilon_{1\,i}$–$\epsilon_{2\,i}$ space. Therefore, the model defined by equations (15) and (16) is coherent

[4] Models of entry have a very similar econometric structure to the analysis of adoption decisions. See Tamer (2003) for a general discussion on incoherence in simultaneous discrete response models.

Figure 2: Adoption of Dichotomous Strategies without Complementarity



and its parameters can be estimated using common methods. Under the assumption that $(\epsilon_{1\,i}, \epsilon_{2\,i})$ are jointly normally distributed, the determinants of the adoption of strategies, $x^{\star}_{1\,i}$ and $x^{\star}_{2\,i}$, can be estimated by means of a standard probit model. Independent estimation of each equation provides consistent estimates of $\boldsymbol{\Theta}$, but taking care of the potential correlation across elements of $\boldsymbol{e}_i$ by jointly estimating both equations increases the efficiency of estimation. Observe however, that we have shown that the existence of correlation across error terms *does not respond* to the existence of complementarity, but rather to co-movements induced by firms' unobserved heterogeneity.

3 Concluding Remarks

This short paper has shown that it is not possible to extend Arora's (1996) reduced form test for the existence of complementarity to evaluate the relationship between a couple of dichotomous strategies. In the case of continuous strategies it could be argued that the correlations among the residuals of the regressions of each strategy on observable firm and market heterogeneity are the result of the existence of complementarities among these strategies and/or the result of unobservable returns to each strategy being positively correlated. We have shown that in the case of dichotomous strategies only the latter case is possible because the existence of complementarities leads to an incoherent simultaneous discrete response model.

References

Arora, A. (1996): "Testing for Complementarities in Reduced–Form Regressions: A Note." *Economics Letters*, 50, 51–55.

Arora, A. and A. Gambardella (1990): "Complementarity and External Linkages: The Strategies of the Large Firms in Biotechnology." *The Journal of Industrial Economics*, 38, 361–379.

Åstebro, T., M. G. Colombo, and R. Seri (2005): "The Diffusion of Complementary Technologies: An Empirical Test." Mimeo, Joseph L. Rotman School of Management, University of Toronto.

Cassiman, B. and R. Veugelers (2006): "In Search of Complementarity in the Innovation Startegy: Internal R & D, Cooperation in R & D, and External Technology Acquisition." *Management Science*, 52, 68–82.

Falk, M. (2006): "Characteristics of Technological and Organizational Innovations." Mimeo, Austrian Institute of Economic Research (WIFO).

Grimpe, C. and K. Hussinger (2008): "Formal and Informal Technology Transfer from Academia to Industry: Complementarity Effects and Innovation Performance." Working Paper 08-080, Zentrum für Europäische Wirtschaftsfforschung (ZEW).

Heckman, J. J. (1978): "Dummy Endogenous Variables in a Simultaneous Equation System." *Econometrica*, 46, 931–959.

Milgrom, P. R. and D. J. Roberts (1990): "The Economics of Modern Manufacturing: Technology, Strategy, and Organization." *American Economic Review*, 80, 511–528.

Miravete, E. J. (2008): "Competing with Menus of Tariff Options." *Journal of the European Economic Association*, 6, forthcoming.

Miravete, E. J. and J. C. Pernías (2006): "Innovation Complementarities and Scale of Production." *Journal of Industrial Economics*, 54, 1–29.

Schmidt, P. (1981): "Constraints on the Parameters in simultaneous Tobit and Probit Models." In C. F. Manski and D. L. McFadden (eds.): *Structural Analysis of Discrete Data with Applications*. Cambridge, MA: MIT Press.

Tamer, E. (2003): "Incomplete Simultaneous Discrete Response Model with Multiple Equilibria." *The Review of Economic Studies*, 70, 147–165.

Vives, X. (2008): "Innovation and Competitive Pressure." *Journal of Industrial Economics*, 56, forthcoming.

Zúñiga, M. P., A. Guzmán, and F. Brown (2007): "Technology Acquisition Strategies in the Pharmaceutical Industry in México." *Comparative Technology Transfer and Society*, 5, 274–296.