Price Discrimination: Theory*

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Abstract

Price discrimination comprises a wide variety of practices aimed to extract rents from base of heterogeneous consumers. When consumer types remain private information and only their distribution is known to the monopolist finding the optimal nonlinear tariff involves solving a constrained variational problem that characterizes the optimal markup for each purchase level so that consumers of different types have no incentive to imitate the behavior of others. Fully separating equilibrium is ensured when the distribution of types fulfills the increasing hazard rate property and individual demands can be unambiguously ranked. Outside this framework, optimal tariffs are difficult to characterize.

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1 Price Discrimination: Theory

A monopolist price discriminates when he sells two identical units of a good at different prices, either to two different buyers, or to the same customer. Two basic elements serve to classify the numerous methods that firms use to price identical units of the product differently: the amount of information available to the seller regarding how different the valuations of consumers are, and the seller's ability to avoid arbitrage. Avoiding arbitrage when firms sell personal services is easy and inexpensive, and thus price discrimination becomes a common practice in such industries. On the contrary, in the absence of restrictions to the transferibility of commodities, low valuation customers could certainly benefit from reselling to higher valuation customers, thus effectively impeding the seller to actually charge two different prices for the product.

Pigou (1922) distinguished between first, second, and third degree price discrimination depending on the amount of information regarding consumers' preferences that is available to the seller. In the case of first degree price discrimination, the seller observes the actual valuation of each consumer and, provided that individual pricing is feasible, he could ask each consumer for her individual reservation price. Individual pricing is however rarely observed in reality but such pricing strategy has the theoretical appeal of leading to the efficient competitive outcome, although obviously with a quite different distribution of rents. This efficiency result vanishes when the seller only knows the distribution of consumers' valuations, as in second degree price discrimination, or when he knows even less —just a signal about consumers' valuations— as in the third degree price discrimination case.

Market segmentation, either geographical or personal may serve as a way to avoid arbitrage. Price differentials across countries are likely to be larger than across neighbourhoods of a city as consumers move more freely in the latter case. Thus, the ability to price discriminate will be partially determined by the importance of consumers' transaction costs to purchase from a different markets. Similarly, the cost of enforcing market segmentation may lead to different pricing schemes. Charging different individuals a different price for a service depending on their location, age, gender, or race is far less expensive from the perspective of monitoring costs than tying prices to the income of each individual. In some circumstances, when third degree price discrimination is used, location, age, gender, race, or any other observable characteristics can be used in an economically efficient (although sometimes morally rotten) way to infer average individual valuations of products and increase profits by extracting a larger share of consumer surplus of those individuals with higher valuations. Thus, in the third degree price discrimination case, solving the price discrimination problem reduces to finding the optimal monopoly price in several independent markets. If there were numerous firms instead of a single seller, the well known inverse elasticity rule should be modified to account for the existence of substitute goods.

More interesting is the case of second degree price discrimination, when the seller needs to avoid the possibility of transferability of demand among consumers of different valuations. Since only the distribution of valuations, and not the valuation of individual consumers is known, finding the optimal pricing scheme requires one to solve a complex problem where the monopolist attempts to extract as much rents as possible from each consumer while at the same time ensuring that they do not imitate the behavior of other consumers with lower valuations. In other words, price discrimination becomes a mechanism design problem where a nonlinear tariff charging a different unit price for each unit sold maximizes the expected profits of the monopolist, while ensuring incentive compatibility.

To solve this problem consumers preferences are assumed to be fully described by $U(q, \theta)$ where q represents the amount of good purchased by a consumer of type θ. This single dimensional index captures the relevant difference in demand of diverse consumers, and leads to non-price related shifts of individual demands. Type θ remains private information for each consumer while the monopolist only knows its distribution $F(\theta)$ on $\Theta = [\underline{\theta}, \overline{\theta}]$. The variational problem that the monopolist faces consists of finding the optimal nonlinear tariff function $T(q)$ that maximizes his expected profits with respect to the distribution $F(\theta)$ provided that in their choices consumers are guided to maximize the net utility $U(q, \theta) - T(q)$. A fully separating equilibrium exists whenever individual demands can be ranked unambiguously, $U_{q\theta}(q, \theta) > 0$, and when the distribution of consumer types $F(\theta)$ fulfills the common increasing hazard rate property (these are sufficient, not necessary conditions). In such a case, the optimal nonlinear tariff $T(q)$ is a concave function leading to quantity discounts that assigns different quantities and payments to consumers of different types. Maskin and Riley (1984) and Mussa and Rosen (1978) (in a framework of quality discrimination) first fully characterize the solution to this canonical version of the price discrimination problem. Contrary to the first degree price discrimination case, now only the highest consumer type, $\overline{\theta}$ is efficiently priced —the efficiency at the top result— while all other consumers are charged a positive markup that induce them to self-select the optimal level of consumption according to the intensity of their preferences, θ. The magnitude of this markup depends on how difficult is to enforce the incentive compatibility condition, which is summarized by the hazard rate of the distribution $F(\theta)$. And the difficulty of separating different consumers depends on how these consumer types are distributed. Thus, the more numerous the consumers with a high valuations are, the larger is the average markup that low valuation consumers should face in order to minimize the incentive of high valuation types to purchase a small amount of the good. Intuitively, the more numerous high valuation consumers are the more likely some of them will attempt to behave as low valuation consumers. To prevent it, a higher markup charged to low valuation consumers is needed in order to reduce sufficiently the outside option of those more numerous high valuation consumers. Consequently, if all consumers are alike, the distribution of consumer types, $F(\theta)$ becomes degenerate, and the optimal nonlinear tariff is a two part tariff with a slope equal to the marginal cost of production and a fixed fee equal to the individual consumer surplus of a buyer.

Engineers —*e.g.*, Dupuit (1849) or Hadley (1885)— rather than economists discovered long ago the advantages of charging different prices to different customers in order to cover the fixed costs of operating transportation services. The solution to the second degree price discrimination problem described above only attracted the attention of economists after the contribution of Mirrlees (1971) in the area of nonlinear taxation. His approach to finding the optimal tax that maximized a social welfare function could easily be adapted to analyze the Ramsey pricing problem of regulated industries contemplated by Ramsey (1927) and Boiteux (1956). With the development of incomplete information games, the nonlinear pricing problem was rapidly reformulated as a direct revelation mechanism —*e.g.*, Goldman, Leland, and Sibley (1984) and Guesnerie and Laffont (1984)—, thus helping out to uncover the technical assumptions that ensured well behaved solutions of the canonical single-product, single-parameters case.

The solution to this canonical price discrimination problem serves as a point of departure for many extensions that have attempted to incorporate either a more general theoretical approach or particular features of specific industries where nonlinear pricing is used to cover fixed costs or to fulfill distributional objectives set by regulators.

A first extension included the possibility that income effect were non-negligible and that consumers could be risk averse. Effectively, this means that the net utility of consumers is not additively separable in payments. Extensions in this direction includes the work of Mirrlees (1976), Roberts (1979), and Wilson (1993, §7).

Another early extension addressed the rationing of stochastic individual demands in the presence of capacity constraints. The nonlinear tariff now attempts to distribute the cost of the installed capacity among consumers according to their usage, as consumers with different loads contribute differently towards the cost of providing the service. But this peak-load pricing solution also provides the firm with incentives to reduce the size of consumers' loads in order to minimize the cost of distributing efficiently the existing capacity among all consumers. Oren, Smith, and Wilson (1985) and Panzar and Sibley (1978) are the two basic references on capacity pricing.

More recently, the basic canonical model of price discrimination has been modified to contemplate the possibility of sequential screening, a process common in many industries where consumers first subscribe to one of the many optional tariffs available and later decide on their optimal level of consumption. The canonical model is modified to allow consumers to learn about their valuation of the product, thus distinguishing between *ex-ante* and *ex-post* types —*i.e.*, the valuation of customers before and after contracting with the seller—, as well as *ex-ante* and *ex-post* incentive compatibility constraints. Courty and Li (2000) consider the case where the *ex-ante* type determines the distribution from where the *ex-post* type will be drawn while Miravete (1996) considers a framework in which the *ex-post* type is the addition of the *ex-ante* type and an independently distributed shock. Both approaches lead to ambiguous results that can somewhat be qualified depending on the stochastic dominance of the composition or convolution distribution, respectively of the *ex-post* relative to the *ex-ante* valuation. Miravete (2005) further evaluates the welfare performance of nonlinear tariff options using data directly linked to consumers *ex-ante* and *ex-post* types.

The most challenging extension of the canonical problem is multidimensional problems. Wilson (1996) presents a concise description of the difficulties that arise when types are multidimensional or the monopolist sells several products. Type dimensions capture different features of consumer demands (intercept, curvature, or others) that independent of prices but are relevant to capture consumer heterogeneity. Multiple products introduce the possibility to account for complementarity and substitution effects and thus design optimal discounts for bundles that include a variable proportion of each good. The difficulty arises because the multidimensional screening problem imposes a continuum of boundary conditions that translate into a large number (as many as type dimensions minus one) of additional partial differential equations that constrain the multidimensional variational problem. Explicit solutions do not exist beyond particular cases such as those studied by Armstrong (1996), Laffont, Maskin, and Rochet (1987), or Wilson (1993, §13-14). A common result reported by Armstrong (1996) and Rochet and Choné (1998) is that low valuation customers are always excluded, thus leading to bunching at the bottom.

Besides the technical difficulties in solving multidimensional price discrimination problems, numerical solutions show that tariffs may become non-monotone and that even the efficiency at the top result may not hold depending on the support of the distribution of types and the interaction among type dimensions given by the specification of the utility function. Perhaps because of these unsurmountable difficulties, the generalized one dimensional nonlinear pricing framework of Rochet and Stole (2002) offer the most promising alternative for advancing in this area of research. Their approach consists of adding a second independent type dimension that enters additively into consumers' utility function. This little modification of the canonical problem disassociates the participation and consumption decisions. While in the canonical problem higher consumer types always participate and purchase more than low valuation customers if lower types participate, now participation is driven by consumer-specific outside options. Now, relative to the canonical price discrimination case, the monopolist loses some ability to extract consumer surplus as profit maximization requires it to balance informational rent extraction from high valuation customers with the participation of low valuation customers.

Characterizing the optimal tariff solution in this model with endogenous participation becomes more involved (although much more feasible) than the general multidimensional case, and it requires solving a two-point boundary problem instead of a simpler recursive first order differential equation with a boundary condition given by the marginal consumer type that decides to participate in the market. Bunching may also occur at the bottom, but only at the bottom, and the tariff is well behaved, continuously approaching the fully efficient solution on the one side and the solution to the canonical pricing problem in the other. Furthermore, the efficiency at the top result survives, and the efficiency at the bottom arises in cases where all consumer types are served.

The model of Rochet and Stole (2002) is also appealing because it offers the possibility to address competitive environments where firms' tariff are the best response to each other's offering and where the tariff offered by the competitor defines the outside option of consumers. This is a model of exclusive agency where consumers subscribe to only one of the firms competing in the industry. The most important result of this literature, also documented by Armstrong and Vickers (2001) is that in industries with full market coverage and where all firms face the same marginal cost, the equilibrium tariff solution is a simple cost plus tariff (Coasian two-part) leading to an efficient allocation of consumption among buyers.

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References

ARMSTRONG, M. (1996): "Multiproduct Nonlinear Pricing." *Econometrica*, 64, 51–75.

ARMSTRONG, M. AND J. VICKERS (2001): "Competitive Price Discrimination." *RAND Journal of Economics*, 32, 579–605.

BOITEUX, M. (1956): "Sur la gestion des monopolies publics astreint à l'équilibre budgétaire." *Econometrica*, 24, 22–40.

COURTY, P. AND H. LI (2000): "Sequential Screening." *Review of Economic Studies*, 67, 697–717.

DUPUIT, J. (1849): "On Tolls and Transport Charges." *International Economic Papers, 2* (translation, 1952), London: Mcmillan.

GOLDMAN, M. B., H. E. LELAND, AND D. S. SIBLEY (1984): "Optimal Nonuniform Pricing." *Review of Economic Studies*, 51, 305–319.

GUESNERIE, R. AND J.-J. LAFFONT (1984): "A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm." *Journal of Public Economics*, 25, 329–369.

HADLEY, A. T. (1885): *Railroad Transportation: Its History and Its Laws*. New York, NY: G. P. Putnam's Sons.

LAFFONT, J.-J., E. MASKIN, AND J.-C. ROCHET (1987): "Optimal Nonlinear Pricing with Two-Dimensional Characteristics." In T. Groves, R. Radner, and S. Reiter (eds.): *Information, Incentives, and Economic Mechanisms*. Minneapolis, MN: University of Minnesota Press.

MASKIN, E. AND J. RILEY (1984): "Monopoly with Incomplete Information." *RAND Journal of Economics*, 15, 171–196.

MIRAVETE, E. J. (1996): "Screening Consumers Through Alternative Pricing Mechanisms." *Journal of Regulatory Economics*, 9, 111–132.

MIRAVETE, E. J. (2005): "The Welfare Performance of Sequential Pricing Mechanisms." *International Economic Review*, 46, forthcoming.

MIRRLEES, J. (1971): "An Exploration in the Theory of Optimum Income Taxation." *Review of Economic Studies*, 38, 175–208.

MIRRLEES, J. (1976): "Optimal Tax Theory: A Synthesis." *Journal of Public Economics*, 6, 327–358.

MUSSA, M. AND S. ROSEN (1978): "Monopoly and Product Quality." *Journal of Economic Theory*, 18, 301–317.

OREN, S. S., S. A. SMITH, AND R. B. WILSON (1985): "Capacity Pricing." *Econometrica*, 53, 545–566.

PANZAR, J. C. AND D. S. SIBLEY (1978): "Public Utility Pricing under Risk: The Case of Self-Rationing." *American Economic Review*, 68, 888–895.

PIGOU, A. C. (1922): *The Economics of Welfare*, 4th edn. London, UK: Macmillan.

RAMSEY, F. P. (1927): "A Contribution to the Theory of Taxation." *Economic Journal*, 37, 47–61.

ROBERTS, K. W. S. (1979): "Welfare Considerations of Nonlinear Pricing." *Economic Journal*, 89, 66–83.

ROCHET, J.-C. AND P. CHONÉ (1998): "Ironing, Sweeping, and Multidimensional Screening." *Econometrica*, 66, 783–826.

ROCHET, J.-C. AND L. A. STOLE (2002): "Nonlinear Pricing with Random Participation." *Review of Economic Studies*, 69, 277–311.

WILSON, R. B. (1993): *Nonlinear Pricing*. New York, NY: Oxford University Press.

WILSON, R. B. (1996): "Nonlinear Pricing and Mechanism Design." In H. Amman, D. A. Kendrick, and J. Rust (eds.): *Handbook of Computational Economics,* Vol. 1. New York, NY: North-Holland.