Time–Consistent Protection with Learning by Doing

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Abstract

Can a government induce efficiency gains in his domestic industry by protecting it against foreign competition? Would such trade protection be time–consistent? The present paper builds a dynamic equilibrium model that accounts for learning–by–doing effects that link firms' strategies over time. The model shows that the existence of dynamic economies of scale suffices to overcome the traditional government's lack of commitment of its tariff policy. This paper compares the infinite horizon Markov Perfect Equilibrium of this game with the dynamic equilibrium under commitment as well as the static Nash equilibrium. Equilibrium strategies are derived in closed form by solving a linear–quadratic differential game. Optimal trade policy involves higher tariff levels than in the static setup in order to account for future gains in efficiency. Under reasonable assumptions, the unique stable MPE is characterized by a domestic price and tariff that decrease as experience accumulates, thus supporting the future liberalization of trade as an equilibrium feature of this dynamic game. JEL: C73, F12, F13.

Keywords: Infant–Industry; Tariff Protection; Infinite Horizon Markov Perfect Equilibria; Linear–Quadratic Differential Games.

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1 Introduction

Infant industry arguments have long been used to justify protectionist trade policies. The essence of such arguments is that local producers must be allowed some time to overcome a temporary disadvantage with respect to foreign competitors. This disadvantage might come from technological backwardness, lack of access to efficient credit markets, local scarcity of the required human capital, or lack of an established reputation. However, the existence of a temporary disadvantage is not sufficient to justify protection. For this, two additional conditions must be satisfied.

First, overcoming the initial handicap must be socially beneficial although not necessarily privately profitable (at least in the short run). This requires some sort of future positive externality to compensate the current welfare loss associated to any protection policies. One possible externality is due to the existence of dynamic economies of scale in the industry. [Corden (1974, §9); Krugman (1984)]. Another type of externality is associated to the existence of experience goods in consumption. Governments could create temporary trade barriers against imports with the argument of defense of diversity through the promotion of the local variety. Thus, temporary protection would generate dynamic efficiency gains through increases in production and/or improved management methods, but also in developing a biased taste for domestically produced goods.

Second, policy intervention must be effective. At least two problems may arise here. One difficulty is that protection policies designed to help domestic producers to become internationally competitive may lead to socially costly collusion between foreign and domestic firms [Gruenspecht, (1988)], or among domestic infant–firms in a protective environment. The second difficulty is that protection should only be granted for the shortest period possible required to make domestic firms competitive. In other words, the government must be able to credibly commit to liberalize trade within a reasonable period of time. Unfortunately, governments can rarely commit credibly to trade policies for more than short periods of time: laws can be changed, treaties can be broken, and government turnover might be high. As pointed out by Matsuyama (1990) this lack of commitment of governments to liberalize trade may explain the persistence of tariff protection. Given the

governments' lack of commitment (political, institutional, or due to lack of reputation), local firms prefer not to become internationally competitive, and given that strategic choice, the best policy for the government is to extend trade protection for some additional period.

This paper presents a framework that identifies a time–consistent tariff protection policy by incorporating the dynamic issues surrounding infant–industries. In doing so I am addressing some common shortcomings in the current treatment of the topic in the trade literature. The basic elements of the model are the following.

1. Tariff Protection Policy: While subsidies or quotas may achieve the same goal of protecting an infant–industry, I choose to study the case of tariff protection policy because this is the instrument most frequently used to protect industries in the early stages of development. Tariff protection was already vindicated by some classic economists, such as List in the 19th Century, as an effective tool to reduce the gap between less developed and industrialized countries.[1]

An additional reason for addressing the case of tariff protection policy is that the model shows how its effectiveness relies on the existence of a taste for variety, which is not required in the case of a subsidy. Thus, protection facilitates local production by introducing a temporary price distortion against imports. An optimally designed tariff should balance the inter–temporal substitution effects of these differentiated goods to the future gains of productivity by domestic firms. It might happen that a strongly biased preference for foreign goods turns socially inefficient any kind of protection unless learning takes place almost instantaneously.

2. Dynamic Economies of Scale: Infant–industries, as envisioned by Protectionists, will benefit from protection merely by having the possibility to produce. Learning by doing is the only source of marginal cost reductions. Additional investments, while possible, are not critical elements of the model. Learning by doing shifts our attention to a framework with truly dynamic strategies where payoffs of different periods are dependent on previous

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[1] The commonly intended superiority of subsidies over tariffs can only be explained because the shadow cost of rising funds in the rest of the economy is unreasonably assumed to be zero.

pricing or production decisions of the domestic firms through the dynamic linkage of its induced marginal cost reductions.

The economic literature has focused frequently in the case where dynamic linkages are not present and thus, investment decisions become independent of the state of the game. For instance in Matsuyama's (1990) model, the domestic firm and the government play a repeated bargaining game where the firm asks for temporary protection in order to develop a cost reducing investment, while the government wants to liberalize trade to maximize welfare. A period later, if the government chose to protect and the firm did not invest, the game remains identical to the one played one period before. However, if learning by doing is considered, there exists at least one state variable, *e.g.*, the level accumulated output, experience, or just the marginal cost, that differs from the previous period due to production. The critical feature of Matsuyama's (1990) model, also present in the work of Miyagiwa and Ohno (1995) and Tornell (1991), is the absence of a dynamic linkage between firms' decisions over time, so that the optimal protection policy becomes time–independent.

By explicitly considering dynamic economies of scale the present approach looses the state–independence of Matsuyama's (1990) model. I therefore require that strategies of the government and the domestic infant–monopolist be dependent on the state of the game. I will therefore restrict my attention to Markov strategies. Focusing in the class of linear–quadratic differential games, the present approach allows me to explicitly characterize an equilibrium where the government's tariff and the monopolist's production schedule are dynamic best responses to each other.

3. Time Consistency: A common criticism to protection policy is the lack of credibility of the government to ending such policy. An example of this lack of commitment to future liberalization is Tornell's (1991) model of *'investment–contingent'* tariffs. In that model, when players reach the supposedly last period of the game, they suddenly become exogenously endowed with asymmetric bargaining power, which makes the government to reconsider its liberalization policy, thus extending protection *'beyond the horizon'* of the game. Two elements of Tornell's (1991) model are worth mentioning here. First,

the government's strategy does not fulfill Markov perfection because at the end of the game protection level is not really determined by past investments of firms whose capital accumulation represents the state of the game, but rather by the firm's threat of increased unemployment. A proper Markov strategy should had accounted for this possibility when computing the investment–contingent protection schedule. Second, the game appears to continue beyond the initial two–period horizon that is so unfortunately common in the trade literature whenever time consistency is questioned. In Tornell's (1991) model, firms actually care for protection beyond the formal finite horizon of the game because their expected payoffs are not restricted to these two periods. Similarly, the government has to care about the future rents of domestic producers when, otherwise it would be optimal from a static point of view to liberalize trade in the supposedly last period of the game.

In order to address the time consistency of the tariff I will focus on Markov strategies defined for infinite horizon games where the possibility of extending protection disappears. This modeling approach may need some justification since it is not common in the trade literature. First, Markov perfection requires that the effect of past actions can be summarized by the state of the game along the equilibrium path. In the present model, the state is represented by the level of marginal cost of the domestic monopolist. Thus, both the monopolist's production schedule and the government's tariff are contingent on this level of marginal cost, whose change leads to a dynamic linkage of strategies over time. Second, players' continuation payoffs at the end of the game should have no effect on the equilibrium tariffs, as it actually happens, for instance, in Tornell (1991), Leahy and Neary (1999), and Miyagiwa and Ohno (1999). In all these models the choice of the horizon length is exogenous and serves the role of simplifying the computation of equilibria. Any subgame perfect equilibria of a finite horizon game is then time–consistent by definition. Restricting the attention to Markov strategies, any Markov Perfect Equilibria (MPE) of a finite T–period game will also be time consistent according to this common view.

I however do not find this definition of time consistency very compelling for problems of tariff protection. A major issue in the game played between the government and firms is always whether firms may convince the government to extend protection beyond the horizon initially announced. Assigning some positive probability to such event implies that

the computation of optimal strategies for a finite horizon of T–periods includes potential continuation payoffs beyond such time horizon T, thus making firms to behave strategically regarding their output or pricing decisions in order to induce an extension of protection that allows them to increase the present value of profits.

The model works as follows. Initially a domestic monopolist faces a very high marginal cost of production that make impossible for her to compete with cost efficient foreign firms that produce a slightly differentiated product. There are however important potential dynamic economies of scale due to learning by doing. The monopolist asks for temporary protection to allow her to reduce her marginal cost and be able to compete once trade is liberalized. The government faces a dynamic trade–off in granting such protection. Consumer surplus will be reduced today due to high cost of domestic production as well as for the high cost of imports induced by the tariff. However, consumer surplus could be enhanced in the future by ensuring the production of the differentiated domestic output and future welfare would be further increased by the profits of a cost efficient domestic industry. The domestic monopolist's pricing decisions over time and the government's schedule of tariffs are made contingent on the marginal cost of the monopolist that captures the state of the game. Strategies are then contingent on the performance of the monopolist through marginal cost reductions induced by pricing and tariff decisions. In equilibrium, both strategies should be the dynamic optimal best response to each other player's strategy. As learning is exhausted, the state converges to its stationary level and price and tariff remain constant from that moment on.

In this framework, the government will credibly bring the level of protection over time up to the stationary level, which may indeed involve an optimal zero tariff, without any need for some exogenous commitment device. This strategy becomes its best dynamic response to the monopolist's also optimal pricing, or equivalently to her production decisions. Intuitively, this result relies on two features of learning by doing mechanisms. Firstly, the marginal social benefit of future learning decreases as the local firm "goes down its learning curve." Secondly, with learning by doing the domestic firm cannot reap the rents from protection without at the same time becoming progressively more efficient: rents are only obtained for positive levels of output once the domestic firm is competitive against

foreign producers, but positive levels of output induce learning and greater efficiency. In other words, learning by doing ties "innovation" and "rent enjoyment" inexorably together.

The paper builds a dynamic equilibrium model of optimal tariff design when learning economies exist and strategies are state contingent. In order to show time–consistency of the equilibrium strategies, the model technical requirements are kept to a minimum. For analytical tractability I use a linear demand specification and learning is assumed to reduce marginal costs. Formally, the model is related to the capital accumulation games of Driskill and McCafferty (1989) and Reynolds (1987), although in the present case the game is not symmetric. The accumulation of experience is translated into a reduction of the monopolist's marginal cost thanks to her own pricing decisions and the tariff decisions of the government. Dynamic considerations lead to domestic prices that are lower or higher than under the static Nash equilibrium depending on the ability of the players to commit to some pricing–tariff schedule. The optimal dynamic tariff is always more protective than myopic static tariff protection policies. The combined effect of higher domestic prices and tariffs induce smaller cost reductions relative to the static equilibrium.

The paper is organized as follows. In section 2, the model and its assumptions are described. Section 3 solves the static Nash equilibrium that ignores any dynamic effects induced by current production decision. Section 4 incorporates such dynamic considerations in a particular framework where both the monopolist and the government can commit to the announced strategies and characterizes a Nash equilibrium in *open–loop* strategies. This section also discusses the validity of such commitment ability. Section 5 characterizes the Markov perfect equilibrium in *closed–loop* strategies that are dynamic optimal best responses so that some external source of commitment to such policy is not needed. Section 6 concludes.

2 Industry Protection with Dynamic Economies of Scale

The game consists of two players: a domestic monopolist and the government of a small country. Initially, foreign firms are much more efficient but the domestic monopolist

enjoys dynamic economies of scale. Therefore, the domestic monopolist asks for temporary protection so that today's sales induce cost reductions and thus allows her to compete in the future once protection is lifted. The government may consider temporary protection because today's reduction in consumer surplus due to high tariffs may get compensated with the increase in monopoly profits, but more importantly because this protection induces a more efficient future provision of the differentiated, domestically produced good.

The model is written in continuous time. The only state variable is the level of marginal cost. Technology is characterized with instantaneous constant returns to scale but marginal cost declines with output due to learning by doing. Current tariff and pricing decisions incorporate an investment component unless learning is exhausted. I also consider the possibility that experience depreciates over time so that some minimum production level is required at every period to ensure a net reduction of the monopolist's marginal cost.

To show that there may exist a time consistent tariff policy and that its existence does not require any external source of commitment, I restrict my attention to the case where both, the monopolist and the government use strategies that are contingent on the state of the game, *i.e.*, the level of marginal cost at each time. The optimal tariff and production schedule is found by characterizing the subgame perfect equilibria in an infinite horizon game where players are restricted to use Markov strategies. The model is solved for the infinite–horizon case to ensure that endpoint transversality conditions are not binding and rule out the possibility that neither the monopolist or the government may be tempted to consider any further extension of protection.

2.1 Demand System

Domestic and foreign products are considered imperfect substitutes by domestic consumers. Let X^t denote the domestic monopolist's production and let M^t denote imports at time t. Assume a quadratic utility function with symmetric cross–effects that is additively separable with respect to money:

$$U[X^t, M^t] = a_x X^t + a_m M^t - \frac{1}{2}\left[b_x (X^t)^2 + b_m (M^t)^2 + 2k X^t M^t\right], \tag{1}$$

where all parameters a_x, a_m, b_x, b_m, k, are strictly positive. The sufficient condition $\Delta = b_x b_m - k^2 > 0$ ensures that the utility function is strictly concave. At each time, t, domestic consumers maximize $U[X^t, M^t]$ subject to the budget constraint:

$$I^t = Q_0^t + P^t X^t + (1 + \tau^t) M^t, \tag{2}$$

where Q_0^t represents the aggregate consumption of a competitive *numeraire* good, and τ^t is the import tariff rate. Foreign firms are competitive and have exhausted learning economies, so that the price of foreign products remain constant at the foreign firms' marginal cost.[2] Since the foreign price of imports only plays a residual role in the model, it has been normalized to one. Thus P^t represents the price of domestic production relative to the price of foreign products before the tariff is applied. The solution of this problem can be written as follows:[3]

$$\begin{bmatrix} a_x - P \\ a_m - (1+\tau) \end{bmatrix} = \begin{bmatrix} b_x & k \\ k & b_m \end{bmatrix} \begin{bmatrix} X \\ M \end{bmatrix}. \tag{3}$$

The demand for domestic and imported goods as functions of their prices and import tariff then becomes:[4]

$$X(P, \tau) = \alpha - \beta P + \gamma(1 + \tau), \tag{4a}$$

$$M(P, \tau) = 1 + \gamma P - (1 + \tau) = \gamma P - \tau, \tag{4b}$$

where $\alpha = (a_x b_m - a_m k)/\Delta > 0$, $\beta = b_m/\Delta > 0$, $\gamma = k/\Delta > 0$. Finally, consumer surplus is given by:

$$CS(P, \tau) = U[X(P, \tau), M(P, \tau)] - P \cdot X(P, \tau) - (1 + \tau) \cdot M(P, \tau). \tag{5}$$

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[2] The assumption of a foreign industry that has exhausted learning is just an extreme case where foreign industries are obviously more developed. But this assumption excludes also the possibility of dynamic strategic effects of current decisions of the domestic firm over foreign firms and *vice versa*, which will surely make the characterization of the solution of the model impossible unless we rely on numerical methods.

[3] Time superscripts are dropped unless their omission may induce ambiguous interpretation.

[4] The intercept and own–price effect of the demand for imports have been normalized to one. In computing equilibria and comparative statics, the values of α and β should be understood as relative magnitudes with respect to the demand for imported products. Technical details and most analytical developments of this and other sections are reported in the Appendix.

Observe that demand does not induce any additional dynamic effect because of its stationary linear specification. Welfare gains from protection might be higher than those highlighted in this model if, in addition, consumption induces dynamic effects on consumers' utility (experience goods), and/or if demand grows over time.

2.2 Marginal Cost Reduction

Technology exhibits instantaneous constant returns to scale but marginal cost, c, is reduced over time while the domestic firm accumulates output. The level of marginal cost is related to the accumulation of experience by the domestic firm. I consider the possibility of depreciation of experience, or in an alternative interpretation, the existence of potential adjustments costs in the accumulation of such experience.[5] The payoff relevant measure of experience is the level of marginal cost. The reduction in marginal cost is described as:[6]

$$\dot{c} = -\lambda[X - \delta c] = -\lambda[(\alpha + \gamma) - \beta P + \gamma\tau - \delta c]. \tag{6}$$

Parameter $\lambda \geq 0$ represents the marginal cost reduction effect per unit of output, while $\delta \geq 0$ captures the idea that the value of experience depreciates over time so that recent output decisions have a stronger effect on the current level of marginal cost than early ones. I will later consider the "limit game" case when $\delta \to 0$.

2.3 The Monopolist's Problem

In an infinite horizon game, the monopolist's problem is to maximize the present value of her profits given the government's tariff, while considering the learning effects induced by current production through her pricing decisions. This problem can be stated as:

$$\max_{P} \int_0^{\infty} \pi(P, \tau, c) e^{-rt} dt \quad \text{s.t.} \quad \dot{c} = -\lambda[X - \delta c] \quad ; \quad c(0) = c^0. \tag{7a}$$

[5] Differential games need of the existence of an adjustment cost to avoid that the dynamic model just consist on the infinite repetition of the corresponding static stage game. This is the role, for instance, of the sticky price assumption in Fershtman and Kamien (1987, 1990), or the depreciation of capital in Reynolds (1987, 1991). The existence of an adjustment cost allows us to identify stationary equilibria in dynamic games.

[6] Observe that marginal cost c, may be increasing if current output does not exceeds δc. This may capture the idea that whenever firms stop investing, they become less competitive.

The instantaneous, constrained, monopoly profits are represented by the following Hamiltonian:

$$H^F = [\alpha - \beta P + \gamma(1+\tau)](P - c - \lambda\mu_f) + \lambda\delta c\mu_f. \tag{7b}$$

2.4 The Government's Problem

The government's problem is to maximize the present value of the sum of consumer surplus, monopoly profits, and tariff revenues, given the monopolist's pricing strategy and accounting for the effects induced by the tariff policy:

$$\max_{\tau} \int_0^{\infty} [CS(P,\tau) + \pi(P,\tau,c) + R(P,\tau)]\, e^{-rt} dt \quad \text{s.t.} \quad \dot{c} = -\lambda[X - \delta c] \quad ; \quad c(0) = c^0, \tag{8a}$$

so that the Hamiltonian associated to the welfare function becomes (see Appendix):

$$\begin{aligned} H^G = {} & \left[\frac{\alpha+\gamma}{\beta-\gamma^2} - c - \lambda\mu_g\right][\alpha - \beta P + \gamma(1+\tau)] + \gamma\frac{\alpha+\gamma}{\beta-\gamma^2}[\gamma P - \tau] + \lambda\delta c\mu_g \\ & - \frac{[\alpha - \beta P + \gamma(1+\tau)]^2}{2(\beta-\gamma^2)} - \frac{\beta[\gamma P - \tau]^2}{2(\beta-\gamma^2)} - \frac{\gamma[\alpha - \beta P + \gamma(1+\tau)][\gamma P - \tau]}{\beta-\gamma^2}, \end{aligned} \tag{8b}$$

where I made use of the tariff revenue function:

$$R(P,\tau) = \tau[\gamma P - \tau]. \tag{9}$$

2.5 The Game

The government has to choose the optimal tariff $\hat{\tau}$ that induces marginal cost reductions through increasing of production of the domestic monopolist, such as it suffices to compensate for the reduction in consumer surplus due to the higher prices paid for imports. Although there are several concepts of equilibrium that I will explore in later sections of the paper, the basic idea is that the tariff has to be the dynamic best response to the firm's pricing strategy and *vice versa*. The optimal level of protection will be conditioned by the potential cost reduction that can be achieved with an additional unit of domestic output.

Obviously this is determined by the level of marginal cost c, which represents the state of the game. Similarly, the firm has to choose the optimal pricing strategy $\hat{P}$ that maximizes the present value of her profits while accounting for the dynamic effects induced by the reduction of her costs as well as the government's tariff policy. Again, the optimal pricing strategy will depend on the level of marginal cost. Consequently, the level of marginal cost in later periods will be determined by the pricing and tariff strategies applied by the monopolist and government respectively.

In games like this one, the state follows a Markov process in the sense that the state of the next period is a function of the current state and actions, and hence, the history at t can be summarized by c^t. To solve this model, I assume perfect information, which implies that both the government and the monopolist know the history of the game, *i.e.*, the previous realizations of the state, c^s, and the vector of control variables, $\{P^s, \tau^s\}$, $\forall s \leq t$. Markov strategies depend only on the state of the system, and players' information sets include only the payoff–relevant history [Maskin and Tirole (1994)]. Markov perfection requires that these strategies are perfect equilibria for any time and state [Fudenberg and Tirole (1986, §2b)]. A differential game equilibrium of this model is a set of functions $\{\hat{P}(c), \hat{\tau}(c)\}$ such that for any time and state, a player's strategy maximizes its payoff from that time on. Applying dynamic programming, the differential game equilibrium of this model solves the following set of generalized Hamilton–Jacobi conditions:[7]

$$H_P^F = \alpha - \beta(2P - c - \lambda\mu_f) + \gamma(1+\tau) = 0, \tag{10a}$$

$$\dot{\mu}_f = (r - \lambda\delta)\mu_f + [\alpha - \beta P + \gamma(1+\tau)] - \gamma[P - c - \lambda\mu_f]\hat{\tau}_c. \tag{10b}$$

Similarly, the optimality conditions for the government are:

$$H_\tau^G = \gamma(P - c - \lambda\mu_g) - \tau = 0, \tag{11a}$$

$$\dot{\mu}_g = (r - \lambda\delta)\mu_g + [\alpha - \beta P + \gamma(1+\tau)] + [\gamma\tau - \beta(P + c + \lambda\mu_g)]\hat{P}_c. \tag{11b}$$

It is useful to write down the dynamic optimality conditions of this system. In order to obtain them, substitute μ_f and μ_g in (10*b*) and (11*b*) from (10*a*) and (11*a*) respectively.

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[7] See for instance Fudenberg and Tirole (1991, §13.3.2) or Vives (1999, §9.2.3).

Differentiate (10a) and (11a) making use of (6) to obtain $\dot{\mu}_f$ and $\dot{\mu}_g$ and substitute the expressions for these time derivatives of the co–state variables into (10b) and (11b). Thus:

$$\begin{bmatrix} (\alpha+\gamma)(r-\lambda\delta+\gamma\lambda\hat{\tau}_c)+(r-2\lambda\delta)\beta c \\ (r-2\lambda\delta)\gamma c \end{bmatrix} = \begin{bmatrix} 2\beta & -\gamma \\ \gamma & -1 \end{bmatrix} \begin{bmatrix} (r-\lambda\delta)P-\dot{P} \\ (r-\lambda\delta)\tau-\dot{\tau} \end{bmatrix} + \lambda \begin{bmatrix} \hat{\tau}_c & 0 \\ 0 & \hat{P}_c \end{bmatrix} \begin{bmatrix} \beta\gamma & -\gamma^2 \\ -2\beta\gamma & \beta+\gamma^2 \end{bmatrix} \begin{bmatrix} P \\ \tau \end{bmatrix}. \tag{12}$$

Evidently, this system of partial differential equations is not easily solved in closed form except for the case of some particular functional specifications such as the linear–quadratic case of the present model. In addition, in dynamic models like this one, the nature of the solution critically depends on the information set that players use. The following sections present a detailed analysis of three alternative equilibrium strategies: static, dynamic with commitment, and dynamic without commitment. This step–by–step approach is useful to isolate the effect of learning as well as to address the issue of commitment and credibility of the announced tariff protection policy.

3 Optimal Static Protection

Suppose that the government and the domestic monopolist play a one–shot game. They choose the tariff and the price of the domestic product that maximizes total welfare and profits respectively. Dynamic considerations about marginal cost are not relevant since the game will not be played in later periods. Co-state variables have no role because the current strategies of players do not affect future payoffs, so that $\mu_f = \dot{\mu}_f = \mu_g = \dot{\mu}_g = 0$. Thus, the static Nash equilibrium (SNE) of this game is found by solving equations (10a) and (11a) while ignoring μ_f and μ_g. Alternatively, solve (12) assuming $\dot{P} = \dot{\tau} = \hat{P}_c = \hat{\tau}_c = \lambda = 0$.

Proposition 1: *The SNE strategies are:*

$$P^N = \frac{\alpha+\gamma+(\beta-\gamma^2)c}{2\beta-\gamma^2}, \tag{13a}$$

$$\tau^N = \frac{\gamma[\alpha+\gamma-\beta c]}{2\beta-\gamma^2}. \tag{13b}$$

Concavity of consumers' utility function ensures that $P^N > 0$. The SNE price increases with α, γ, and c, *i.e.*, with positive non–price related shifts of demand, the domestic firm's marginal cost, and the degree of substitution between the domestic and foreign product. The SNE tariff increases with α but decreases with β and c. The SNE also increases with γ. The closer substitutes are imports to domestic production, the higher the tariff protection needed to ensure the production of the domestic variety.

Condition $\alpha + \gamma - \beta c \geq 0$ ensures not only that the tariff is positive, but also that the domestic firm prices above the marginal cost. If marginal cost c is so high relative to the demand parameters that only allows the domestic firm to sell its production by pricing below the marginal cost, then we have reached a corner solution where the domestic monopolist does not produce and the government has no interest in establishing a tariff on imports. Since the problem is static, $\dot{c} = 0$. Evaluating the outcome at the steady state level of output δc given by equation (6), and making use of of the static Nash solution (13), the associated level of marginal cost in this static equilibrium is:

$$c^N = \frac{(\alpha + \gamma)\beta}{(2\beta - \gamma^2)\delta + \beta^2}. \tag{14}$$

This SNE ignores, by definition, all dynamic effects that link current production to future cost reductions. The solution presented here will be useful to compare the equilibrium impact of these dynamic effects for different informational assumptions.

4 Committing to Protect and Produce

In this section we turn our attention to dynamic strategies. I first focus in situations in which *both* the government and the domestic monopolist have the ability to commit to some strategy announced at the initial state of the game, leading to an *open–loop* equilibrium (OLE). This section characterizes such equilibrium and compares it to the SNE. I also discuss the validity of this commitment assumption.

An OLE is a Nash equilibrium in *open–loop* strategies. In practice this means that at $t = 0$ the firm announces a schedule of prices, $\{P^s\}_{s=0}^{s=\infty}$, while the government

announces a schedule of tariffs, $\{\tau^s\}_{s=0}^{s=\infty}$ for the infinite horizon of the game. The game is only dynamic because it involves price and tariff decisions over the whole horizon of the game, but truly dynamic interactions are omitted because both schedules are one–shot decisions announced at the beginning of the game. The concept of OLE assumes that both firm and government can commit to carry the announced strategies regardless of how the game evolves in the future. This is actually the source of most time–consistency problems that characterizes many dynamic economic models.[8]

Since the game involves several periods, players will only account for the effect of their own control strategies on the state of the game. Thus, co–state variables μ_f and μ_g now appear in the objective function of the firm and the government respectively. Since they account for payoff consequences of changes in the state, the effect of H_c^F and H_c^G on the co-state variables μ_f and μ_g are also considered. But because the announcement is made at the beginning of the game, players cannot take into account any feedback effect of the decisions of their opponents. Thus, the stationary OLE solves (12), although making $\dot{P} = \dot{\tau} = \hat{P}_c = \hat{\tau}_c = 0$.

PROPOSITION 2: *The stationary OLE strategies are:*

$$P^\circ = \frac{(\alpha+\gamma)(r-\lambda\delta)+(r-2\lambda\delta)(\beta-\gamma^2)c}{(r-\lambda\delta)(2\beta-\gamma^2)}, \tag{15a}$$

$$\tau^\circ = \frac{\gamma[(\alpha+\gamma)(r-\lambda\delta)-(r-2\lambda\delta)\beta c]}{(r-\lambda\delta)(2\beta-\gamma^2)}. \tag{15b}$$

PROPOSITION 3: *The stationary OLE is globally stable if:* [9]

$$r < \lambda\delta. \tag{16}$$

PROPOSITION 4: *The stationary OLE coincides with the SNE whenever $\lambda=0$, $\delta=0$, or $c=0$.*

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[8] In addition to the trade literature discussed in the Introduction, see for instance Miyagiwa and Ohno (1999), Pearce and Stacchetti (1997), and Staiger and Tabellini (1987).

[9] Observe that one implication of this stability condition is that P° is increasing in c, similar to the behavior of P^N. The derivative of τ° with respect to c is also negative, similarly to the case of τ^N.

If $c=0$ no further cost reductions are possible, and the game becomes necessarily static. When $\lambda=0$ there is no dynamic linkage between current production and marginal cost reduction, and thus the model also becomes static. When $\delta=0$ experience does not depreciates over time. Therefore, it is not necessary to produce any output just to maintain the same level of marginal cost of the previous period. The combined effect of these two dynamic elements leads to the following result.

PROPOSITION 5: *In the stationary OLE, the equilibrium tariff exceeds the SNE tariff, but the equilibrium price is lower than in the SNE case:*

$$P^{\circ} - P^{N} = \frac{\lambda\delta(\beta-\gamma^2)c}{(r-\lambda\delta)(2\beta-\gamma^2)} < 0, \tag{17a}$$

$$\tau^{\circ} - \tau^{N} = \frac{-\gamma\lambda\delta\beta c}{(r-\lambda\delta)(2\beta-\gamma^2)} > 0. \tag{17b}$$

Therefore, the monopolist has a stronger dynamic incentive to underprice (overproduce) relative to the static setting the less substitutable domestic product and imports are (low γ). This optimal dynamic strategy increases the present value of the marginal revenue of the monopolist while minimizing the investment in marginal cost reduction.

Since the monopolist does not internalize the reduction in consumer surplus due to the high price of the domestic product, the optimal strategy for the government involves a higher tariff than in the static case. Sometimes, if the initial marginal costs are high enough, only if the government intervenes the monopolist will produce. In intermediate cases, government protection will maximize total welfare by speeding up the process of learning by doing by the domestic producer. By being more protective, the government ensures some additional market power to the domestic monopolist to induce further cost reductions that later compensate the current reduction in consumer surplus due to high prices of the domestic product. Observe that the increase in the optimal OLE relative to the SNE tariff is more important the higher the cost of the domestic monopolist c, the more elastic is the demand for the domestic product (large β), or the less substitutable domestic and foreign goods are (small γ).

The combined effect of these equilibrium pricing and tariff strategies is to lower the stationary equilibrium level of the monopolist's marginal cost relative to the SNE. This

is a direct consequence of the monopolist and the government accounting for the future savings induced by their pricing and tariff decisions.

PROPOSITION 6: *In the stable stationary OLE, the domestic monopolist's marginal cost is positive but below the level in the SNE:*

$$0 < c^{\circ} = \frac{(\alpha+\gamma)(r-\lambda\delta)\beta}{(r-\lambda\delta)(2\beta-\gamma^2)\delta+(r-2\lambda\delta)\beta^2} < c^N. \tag{18}$$

Both c^N and c° are increasing in α and γ but decreasing in β and δ. Observe however that the difference among them disappears when $\delta = 0$ even if learning effects are significant, *i.e.*, $\lambda > 0$. This "limit game" without adjustment cost turns into an infinitely repeated static game. This result is important because if we ignore the depreciation of experience, dynamic models would lead to exactly the same steady state level of marginal cost in the long run. However, the more important is the effect of the depreciation of experience, the lower the stationary equilibrium level of the marginal cost would be relative to the static equilibrium.

4.1 Is Commitment Credible?

Frequently, trade models find that governments have no dynamic incentives to reduce protection. These models, *e.g.*, Miyagiwa and Ohno (1995), only address the case of homogeneous products. The present model replicates these results for the case of independent goods, $\gamma = 0$, and when the stationary *open–loop* tariff coincides with the optimal static one. This result is important because the present model also shows that if domestic and foreign products are slightly differentiated, then governments will more likely consider dynamic effects, and therefore they will increase tariff protection relative to the static case to successfully induce marginal cost reductions by the domestic producer. This is a consideration that has so far been ignored in the trade literature.

The comparison between OLE and SNE of equations $(17a) - (17b)$ also illustrates a common criticism to protection policies. In general, the domestic monopolist is now more aggressive than in the static framework, charging lower prices and speeding up cost

reduction. The government's optimal tariff policy is now more protective than in the static case. The government has to make imports less attractive the higher is the initial disadvantage of the domestic firm. This policy might be very damaging to consumers who bear the cost of protection by facing higher domestic prices of imports as well as higher prices of domestically produced goods until significant learning has been realized. But this policy also ensures that only the minimum additional market power is granted to the domestic monopolist to effectively induce cost reductions while minimizing the combined loss of consumer surplus due to high prices of domestic good and imports, while allowing the monopolist to keep producing, and therefore further reducing her marginal cost.

Will this protection policy be actually enforced? Most likely not. The monopolist and the government receive feedback information as the game evolves. They do not use this information only because of the way *open–loop* strategies are defined. But it is not reasonable that they do not make use of such information in a later state of the game if it is in their own interest to do so. The government may find that the monopolist is reducing her cost very slowly. A low tariff or an early liberalization of trade may put in danger the survival of the domestic production and jeopardize the whole intention of the protection policy. Thus, the government will most likely deviate from the announced tariff schedule either by increasing the import tariff or postponing trade liberalization. Similarly, the monopolist may find that the government's tariff is lower than expected and that it does not ensure her survival in the long run. The monopolist then deviates from her announced strategy by increasing her price to maximize the rent extraction while protection lasts.[10]

According to Reynolds (1987, §3), there are two main reasons why we can consider *open–loop* strategies. First, *open–loop* strategies are reasonable when players are unable to observe the state affecting decisions of the other players after the beginning of the

[10] These are the typical cases commonly documented in the theoretical and trade policy literature. However, they are not the only possible deviations from an OLE. At least in theory, it is also possible that the government discovers that his tariff is too high, thus reducing consumer surplus too much while allowing the domestic monopolist an excessive market power. The government may then find optimal to modify his tariff policy in order to reduce the inefficiency of the announced tariff. Similarly, the monopolist may find that the announced tariff is so high that it shifts demand for the domestic good beyond her expectations. In such a case, the monopolist will also find optimal to deviate from the announced pricing schedule to lower her marginal cost and thus increase her profit margin as much and as long as possible. The lack of commitment is also present in these two latter cases.

game. This is not the case in the present model. The monopolist always observes her marginal cost and the government, even when he cannot observe it directly, can infer it with certainty from the monopolist's price and the known parameters of demand and marginal cost dynamics.

The second reason given by Reynolds is that the rules of the game may require precommitment to *open–loop* strategies. This is the case of trade agreements, multilateral agreements through the WTO, and even the passing of constitutional amendments. All these mechanisms are ways to signal that the government is going to commit to certain trade policy, perhaps leading to trade liberalization in the future. But, as pointed out by the examples discussed before, *open–loop* strategies are not dynamic best responses to each other player's strategy because they were decided at the beginning of the game and neglect, by definition, any potential feedback effect, *i.e.*, any payoff relevant action of the other player that may have an unexpected effect on the evolution of the game.

If the difference between the expected and actual payoffs compensates potentially important transaction or reputation costs, the government will break any trade agreement or modify any necessary constitutional amendments, thus easily turning tariff protection permanent. This is the result systematically reported in the trade literature using time–independent strategies in models without dynamic linkage of payoffs over time, *e.g.*, Miyagiwa and Ohno (1995) and Tornell (1991). This section has shown that using state–contingent strategies that account *only* for each player's own–induced dynamic effects is not sufficient to avoid time–consistency problems. The next section studies how to characterize a tariff protection policy that is robust to the time–inconsistency criticism.

5 Optimal Dynamic Protection

In this section I address the solution of the model when both players take into account not only the dynamic effects induced by their own strategies, but also recognize the dynamic effects induced by the other player's strategy. Thus, $\hat{P}_c$ and $\hat{\tau}_c$ in equation (12) are not assumed zero anymore. Now the monopolist's pricing and the government's tariff strategies

are going to be made explicit functions of the state of the game c, *i.e.*, we will use Markov strategies.

Contrary to the OLE solution of the previous section, the sequence of prices and tariffs that characterize the *closed–loop* solution cannot be announced at the initial stage of the game. Neither the government or the monopolist need to commit to a predetermined sequence of actions, as they will optimally choose their strategies at each state of the game. An MPE is just a subgame perfect equilibrium in Markov strategies. Thus, players take into account each other actions and the equilibrium strategies have to be optimal from any time t onwards. Therefore, the equilibrium strategies are by definition time–consistent.[11]

Solving for MPE is analytically much more involved than characterizing SNE or OLE strategies. Since the model is not symmetric, solving for MPE requires to find the intersections among the two hyperbolas that represent the system of four nonlinear optimality conditions. There are two approaches to deal with such difficult problem: i) Solve the model numerically, find the multiple solutions and identify stable equilibria; ii) Analyze how does the solution depends on the parameters of the model.

The first approach is valid if we want to evaluate the model for a given set of parameters, perhaps corresponding to a particular calibration or estimation of demands, costs, and learning effect parameters. This approach however lacks generality and although several sensitivity exercises may confirm the robustness of the features of the found MPE, it is not easy to identify the effect of each parameter on the stability and/or uniqueness of the solution because both result from nonlinear interactions among the parameters.

I will follow the second approach in order to study, among other issues, whether an MPE exists for more than a single combination of parameters, thus making the existence and uniqueness result of more general interest. The drawback of this approach is that the resulting conditions are only valid "asymptotically." The nonlinear relations among

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[11] I should mention that under restrictive conditions, the OLE discussed in the previous section coincides with the MPE, and thus, following the announced strategies will turn out to be the dynamically optimal, time–consistent, equilibrium strategies. See for instance Reinganum (1982b). This is not the case in the present model because the necessary conditions for the OLE are not independent of the state variable c. Therefore, OLE will never coincide with the time–consistent MPE. See Fershtman (1987) as well as Başar and Olsder (1995, §5.6).

parameters characterized by the hyperbolas that define the equilibrium conditions are replaced by their corresponding asymptotes, thus characterizing the equilibria through sufficient conditions that depend explicitly on the parameters of the model.

In what follows, I describe how are the relevant parameters computed. I do not characterize every single parameter of the model, but only those who drive its dynamic features, *i.e.*, , $\hat{P}_c = \phi_2^f$ and $\hat{\tau}_c = \phi_2^g$ in equation (12).

5.1 Computing Linear Markov Perfect Equilibria

I now solve for MPE. Markov strategies are state dependent and therefore embody the idea of a protection policy that is contingent on the industry's performance. The equilibrium is one of simultaneous moves of both players at each time and state.[12] In equilibrium, the government's optimal tariff policy is the optimal dynamic best response to the monopolist's pricing strategy and *vice versa*. Let $P^\star(c) \in S^F$ denote the monopolist's Markov pricing strategy out of the set of all possible state contingent pricing strategies S^F. Similarly, $\tau^\star(c) \in S^G$ is the government's Markov tariff strategy out of the set of all possible state contingent tariff strategies. The MPE is a pair of strategies $\{P^\star(c), \tau^\star(c)\} \in S^F \times S^G$ that maximizes each player's value function for any state c given the equilibrium Markov strategy of the other player. The value functions of this game for the monopolist and the government are:

$$V^F[c(s)] = \int_s^\infty \pi[P, \tau, c(t)]\, e^{-rt} dt, \tag{19a}$$

$$V^G[c(s)] = \int_s^\infty W[P, \tau, c(t)]\, e^{-rt} dt, \tag{19b}$$

[12] I do not consider the possible role of the government as Stackelberg leader, neither with total or instantaneous pre–commitment as defined and studied by Cohen and Michel (1988). If the government alone decides at the beginning of the game to set some course of action, *i.e.*, the tariff schedule, regardless of the actions of the other player, that is, ignoring any potential feedback from the actions of the other player, then he is following an *open–loop* Stackelberg strategy that violates the Bellman principle of optimality, and it is therefore time–inconsistent.

where the marginal cost evolves according to equation (6). Starr and Ho (1969) show that the following pair of Bellman equations provides with a set of necessary conditions for MPE strategies:

$$rV^F(c) = \max_{P\in S^F} \left\{ \pi(P,\tau,c) - \left(\frac{\partial X}{\partial P} + \frac{\partial X}{\partial \tau} \right) \dot{c} V^F \right\}, \tag{20a}$$

$$rV^G(c) = \max_{\tau\in S^G} \left\{ W(P,\tau,c) - \left(\frac{\partial X}{\partial P} + \frac{\partial X}{\partial \tau} \right) \dot{c} V^G \right\}. \tag{20b}$$

After some algebra the first order conditions for maximizing the terms between curly brackets in (19) become (see Appendix):

$$0 = \alpha - \beta[2P - c + \lambda(\beta - \gamma)V_c^F(c)] + \gamma(1+\tau), \tag{21a}$$

$$0 = \gamma[P - c + \lambda(\beta - \gamma)V_c^G(c)] - \tau. \tag{21b}$$

Solving this system of first order conditions we can write the optimal strategies as reduced form functions of the state c.[13] The optimal state contingent price and tariff strategies are:

$$P^\star(c) = P^N(c) - \frac{\lambda(\gamma-\beta)[\beta V_c^F(c) - \gamma^2 V_c^G(c)]}{2\beta - \gamma^2}, \tag{22a}$$

$$\tau^\star(c) = \tau^N(c) - \frac{\lambda\beta\gamma(\gamma-\beta)[V_c^F(c) - 2V_c^G(c)]}{2\beta - \gamma^2}. \tag{22b}$$

Equations $(22a)$ – $(22b)$ reveal several interesting features of the MPE strategies. First note that the MPE coincides with the SNE when $\lambda = 0$, *i.e.*, there are no potential cost reductions due to learning by doing and neither monopoly pricing or a protective tariff can induce any dynamic effect whenever current production does not carry any investment consideration. We thus need the existence of some dynamic economy of scale to derive equilibrium strategies that differ from the static ones.

Second, MPE and SNE also coincide when $\beta = \gamma$, *i.e.*, when the own and cross–price effects are the same. But even in this case, the inequality $\beta - \gamma^2 > 0$ must be fulfilled to

[13] The solution to these Bellman equations is subgame perfect because they hold for any state c.

ensure the concavity of the utility function of domestic consumers. Combining these two conditions the equality of own and cross–price effects may hold for $\gamma = \beta < 1$. As β and γ approach 1 the domestic and the imported good becomes almost perfect substitutes [Vives (1999, §6.1)], and dynamic considerations are no longer valid because consumers can buy an "identical" foreign product. In this case, the consumer surplus effect outweights any potential future savings through reduction in the marginal cost. This means that the mere future increase in domestic profits does not suffice to compensate the current reduction in consumer surplus and that the infant–industry argument will not justify protection of the domestic industry unless such protection also leads to the production of a differentiated domestic variety.

Third, in addition to these effects, observe that, as before in the OLE, the optimal MPE tariff also coincides with the SNE tariff when $\gamma = 0$, *i.e.*, when domestic and foreign products have independent demands.

Finally, notice that necessary conditions $(21a) - (21b)$ are sufficient for a maximum because the expressions in curly brackets in $(20a) - (20b)$ are concave in $\{P, \tau\}$ as long as the utility function of domestic consumers is concave, *i.e.*, $\beta - \gamma^2 > 0$.

Substitution of $(22a) - (22b)$ into $(20a) - (20b)$ produces a system of two partial differential equations that is difficult to solve in general. However, since this is a linear–quadratic differential game, it is reasonable to assume quadratic value functions in the state in order to solve this problem:

$$V^F(c) = \psi_0^f + \psi_1^f c + \frac{\psi_2^f}{2} c^2, \tag{23a}$$

$$V^G(c) = \psi_0^g + \psi_1^g c + \frac{\psi_2^g}{2} c^2. \tag{23b}$$

One way to proceed is to substitute the quadratic value functions into the Bellman equations $(20a) - (20b)$ and differentiate the expressions between curly brackets. The resulting system of six nonlinear equations in the value function parameters has to be satisfied for any state c. Fortunately this system is block–recursive. Only two nonlinear equations determine $\{\psi_2^f, \psi_2^g\}$. Once these parameters have been found, another two

nonlinear equations determine $\{\psi_1^f, \psi_1^g\}$. Finally, the remaining two equations determine $\{\psi_0^f, \psi_0^g\}$. Observe however that after substituting $(23a) - (23b)$ into $(20a) - (20b)$, the quadratic specification for the value functions leads to Markov strategies $(22a) - (22b)$ that are linear in the state. Without loss of generality we can write these strategies as follows:[14]

$$\hat{P}(c) = \phi_1^f + \phi_2^f c, \tag{24a}$$

$$\hat{\tau}(c) = \phi_1^g + \phi_2^g c. \tag{24b}$$

Given the block–recursive system of nonlinear equations in the value function parameters, an alternative but equivalent approach is to solve the two coupled Riccati equations associated to $(24a) - (24b)$ and the optimality conditions $(10) - (11)$. The procedure is to substitute $(24a) - (24b)$ in (12), as well as their derivatives $\hat{P}_c = \phi_2^f$ and $\hat{\tau}_c = \phi_2^g$. Assuming an stationary MPE, impose $\dot{P} = \dot{\tau} = 0$ and equate coefficients of the intercept and slope of these strategies to obtain the following Riccati equations:

$$\begin{bmatrix} A_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2\beta & -\gamma \\ \gamma & -1 \end{bmatrix} \begin{bmatrix} (r-\lambda\delta)\phi_1^f \\ (r-\lambda\delta)\phi_1^g \end{bmatrix} + \lambda \begin{bmatrix} \phi_2^g & 0 \\ 0 & \phi_2^f \end{bmatrix} \begin{bmatrix} \beta\gamma & -\gamma^2 \\ -2\beta\gamma & \beta+\gamma^2 \end{bmatrix} \begin{bmatrix} \phi_1^f \\ \phi_1^g \end{bmatrix}, \tag{25a}$$

$$\begin{bmatrix} A_2\beta \\ A_2\gamma \end{bmatrix} = \begin{bmatrix} 2\beta & -\gamma \\ \gamma & -1 \end{bmatrix} \begin{bmatrix} (r-\lambda\delta)\phi_2^f \\ (r-\lambda\delta)\phi_2^g \end{bmatrix} + \lambda \begin{bmatrix} \phi_2^g & 0 \\ 0 & \phi_2^f \end{bmatrix} \begin{bmatrix} \beta\gamma & -\gamma^2 \\ -2\beta\gamma & \beta+\gamma^2 \end{bmatrix} \begin{bmatrix} \phi_2^f \\ \phi_2^g \end{bmatrix}, \tag{25b}$$

where $A_1 = (\alpha+\gamma)(r - \lambda\delta + \gamma\lambda\phi_2^g)$ and $A_2 = r - 2\lambda\delta$. Riccati equations are independent of c because MPE should hold for any realization of the state. Furthermore, observe that $\{\phi_1^f, \phi_1^g\}$ can be found as a linear combination of the equilibrium values of $\{\phi_2^f, \phi_2^g\}$. Therefore, we can focus in solving the simpler system of nonlinear equations $(25b)$, whose parameters $\{\phi_2^f, \phi_2^g\}$ characterize the dynamic features of the equilibrium strategies.[15]

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[14] I show in the Appendix that parameters ψ's can be written as a linear combination of parameters ϕ's. Since the Bellman equations are concave in $\{P, \tau\}$, these linear Markov strategies lead to Bellman equations that are also concave in c. Therefore, focusing on the linear first order conditions suffices to characterize the maximizing strategies.

[15] Riccati equations do not identify $\{\psi_0^f, \psi_0^g\}$ directly, *i.e.*, the parameters related to the intercept of the value function. I will ignore these parameters because they are not relevant for the stability perfection of the equilibria discussed later in the paper. The quadratic value function approach is fully described in Driskill and McCafferty (1989) and Reynolds (1987). Jun and Vives (1999, §4.2) discuss the equivalence between the quadratic value function and the linear Markov strategy approach followed here.

5.2 Stationary MPE

The existence of cross–products and quadratic terms in the Riccati equations opens the possibility of multiple solutions. Actually, the Riccati equations are hyperbolas on $\{\beta, \gamma\}$ for any given set $\{r, \lambda\, d, \alpha, c\}$. Figure 1 summarize the features of this equilibrium for a well behaved set of parameters.[16] Figure 1 shows the four intersections of the *'Riccati Hyperbolas.'* But instead of solving numerically the highly nonlinear equations in the model's parameters that characterize the couple of hyperbolas defined in (25*b*), I focus on the asymptotes of these hyperbolas (also shown in Figure 1) to obtain some qualitative results that can be easily linked to the parameters of the model. I first characterize the stability of the MPE of this model. I then explore sufficient conditions for the existence of a unique stable MPE.

Proposition 7: *A pair of Markov strategies* $\{\hat{P}(c), \hat{\tau}(c)\}$ *yields a stable marginal cost trajectory whenever:*

$$\beta\phi_2^f - \gamma\phi_2^g + \delta < 0. \tag{26}$$

Proposition 8: *If* $\beta > \gamma$*, and* $\gamma > \beta/[2(1-\beta)]$*, then* $\exists\delta^\star$ *such that* $\forall\delta \geq \delta^\star$*, there is a unique stable MPE.*

While $\beta - \gamma^2 > 0$ ensures the concavity of the utility function (1), the more restrictive condition $\beta - \gamma > 0$ requires that own–price effects always exceed cross–price effects between imports and domestic production. Thus, we exclude cases where domestic customers have a strongly biased taste for foreign goods. The second condition requires that imports are closer substitutes to domestic production the more elastic (larger β) is the demand for the latter. Equations (13*b*), (17*b*), and (22*b*) have pointed out in different environments that a tariff is not effective if imports and domestic products are independent. Proposition 8 requires that they are "close enough" substitutes so that establishing a tariff on imports

[16] In particular $\alpha = 1$; $\beta = 0.4$; $\gamma = 0.35$; $\lambda = 0.7$; $\delta = 0.13$; and $r = 0.05$ so that the concavity, $\beta > \gamma^2$, and the OLE stability, $r < \lambda\delta$, conditions hold. These parameters also fulfill all the requirements for a single stable MPE stated below in Proposition 8.

induces a significant increase in demand that trigger marginal cost reductions for the domestic monopolist.[17]

A major result that follows from Proposition 8 is that the Markov Perfect domestic pricing and tariff of equation (24) are both decreasing with the state as $\hat{P}_c = \phi_2^f < 0$ and $\hat{\tau}_c = \phi_2^g < 0$. This is exactly what we should expect from a model of learning. Tariff protection initially shifts the demand in favor of domestic production. As learning takes place, the domestic monopolist is able to offer these products at lower prices, and thus protection does not need to remain as high as before, as the positive price effect compensates the negative demand effect of a less protective policy. Thus, the time–consistent tariff policy leads to future liberalization of trade. Notice that the parameters of the model determine the steady state level of the marginal cost of the domestic monopolist. Whether this liberalization is full or not depends exclusively on the relative magnitude of the marginal costs of the domestic monopolist and that of foreign producers, which again is given by the parameters of the model.

5.3 Results from Simulations

To complete the analysis I should compare the optimal MPE pricing and tariff strategies, as well as of the level of marginal cost, at the steady–state relative to those of the OLE. The infinite–horizon MPE of this model does not suffer from any weakness related to the possibility of extension of the game beyond some given finite horizon . Equilibrium strategies are dynamic best response to each other player's and do not need of any external source of commitment. Therefore, comparing these strategies to OLE strategies, we can account for the effect of the lack of commitment of the domestic firm and the government on their actual strategies.

[17] Notice that both conditions will only be simultaneously satisfied if $\beta < 0.5$. It should be stressed that these conditions limit the interactions of control and state variables in players' payoff functions, as well as those of the square of the opponent's control on each player payoff functions. This can be easily shown by analyzing cross–products in the instantaneous payoff functions associated to (7b) and (8b). Lockwood (1996), has shown that these are sufficient conditions for uniqueness of linear infinite horizon MPE.

To compare the steady–states of the MPE and the OLE, just assume $\dot{P} = \dot{\tau} = 0$ in equation (12). Obviously when $\hat{P}_c \to 0$ and $\hat{\tau} \to 0$, MPE approaches OLE given by equations $(15a) - (15b)$. Unfortunately, as we have seen previously, $\hat{P}_c = \phi_2^f$ and $\hat{\tau}_c = \phi_2^g$ are the solution to a set of nonlinear equations on the parameters of the model. This requires computing the unique stable equilibria of Figure 1 several times to account for the following results, that are therefore here only based in numerical simulations and summarized in Figures 2–4.[18]

Figure 2 shows that $P^\star > P^\circ$. The lack of commitment ability makes the monopolist to behave less aggressive in inducing cost reductions through learning by doing. Actually, $P^\star$ also exceeds P^N. Therefore, the monopolist will only price aggressively to induce learning if she and the government are able to commit to a particular pricing–tariff schedule. These differences in pricing strategies are more important the larger is β, *i.e.*, the more elastic is the demand for the domestic product, but γ appears not to be any definite effect. Figure 3 shows that $\tau^\star > \tau^\circ$, that is, the optimal MPE tariff strategy is always more protective than in any of the other previously studied environments. Observe that these differences are both increasing in β and γ. Finally, Figure 4 shows that the stable MPE involves higher levels of marginal costs than in the OLE case, a relation that is also increasing in β and γ.

6 Concluding Remarks

This paper presents a major general result and several characterizations of the pricing–tariff dynamic equilibrium when the domestic monopolist reduces her marginal cost through learning by doing and the government designs a tariff to maximize the discounted value of total domestic welfare, thus accounting for future gains induced by current domestic production decisions.

[18] For the simulations, solutions are evaluated in a range of $\pm 10\%$ the value of β. Similarly, γ varies between 10% and 90% of the range defined by the two conditions of Proposition 8 for each particular value of β. Marginal cost c is assumed to be 3.5, a value slightly above the optimal Nash pricing of 3.375 for this set of parameters. This is the interesting case because domestic production will not even start if prices do not cover marginal costs.

The major result is that contrary to many existing models in the trade literature, the present paper shows that it is possible to characterize a time–consistent tariff protection policy that successfully help the domestic infant–industry become internationally competitive. This is a key question in classical trade theory. The model shows that it is not necessary any external source of commitment to avoid future deviations from this policy. The existence of learning effects makes possible to find an equilibrium in Markov strategies where the government's tariff is the dynamic best response to the domestic monopolist's pricing decisions and *vice versa*. Two modeling choices, also absent in the existing literature dealing with time consistency of tariff protection policies, are critical to show this dynamic optimality result: the use of truly state contingent strategies, and solving the infinite horizon version of the game to explicitly avoid the possibility of extending the game beyond any initial finite horizon. Furthermore, the analysis of the stability of equilibria shows that MPE leads to future liberalization of trade.

The existence of learning by doing could induce overproduction relative to the static equilibrium, thus maximizing the total discounted profits by lowering the marginal cost. However, this result only holds when both the monopolist and the government can commit to a particular schedule of pricing and tariff decisions. To circumvent the lack of commitment of OLE strategies, I characterize the infinite–horizon linear MPE strategies. Since neither the monopolist or the government are able to commit to a specific schedule of actions, the resulting dynamic equilibrium is more damaging for consumers: both the domestic prices and tariffs are higher, although marginal costs reach levels lower than in the OLE case when domestic and imported production are close substitutes. Thus, even with higher domestic prices, the monopolist overproduces because the higher tariff shifts demand significantly from imports to domestic goods.

The model also shows that in the absence of depreciation of experience, the solution of the dynamic OLE mimics the static one, and thus infant–industry tariff protection fails to reduce marginal cost of domestic firms. Similarly, if the domestic and imported good are considered independent, the dynamic OLE and the static equilibrium strategies coincide and infant–industry arguments will also fail.

Finally, something must be said about potential policy implementations of this model. Instead of considering uncertain learning effects, as in Dinopoulos, Lewis, and Sappington (1995), the model assumes perfect information regarding all parameters of the model, including the learning equation. It could be argued that such demanding informational requirement makes the application of the model impractical. In addition, and despite the tedious and complex computations needed to characterize the equilibrium, results are contingent on the specific linear–quadratic structure of this model. While recognizing that both assertions carry some truth, I should emphasize that linear–quadratic differential games are commonly interpreted as a first approximation to MPEs of more complex differential games, for whom closed–form solutions are impractical [Reinganum (1982a, §1)]. Furthermore, the linear MPE analized in this paper is robust to zero–mean additive shocks in the learning equation (6) [Vives (1999, §9.2.3)], thus making the informational requirement argument a less striking criticism.

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Appendix 1

• Demand System

In order to reduce the number of parameters of the model, the specification of demand normalizes some of them. From equation (3), the demand for domestic and imported products are:

$$X(P,\tau) = \frac{(a_x b_m - a_m k) - b_m P + k(1+\tau)}{b_x b_m - k^2}, \tag{A.1}$$

$$M(P,\tau) = \frac{(a_m b_x - a_x k) + kP - b_x(1+\tau)}{b_x b_m - k^2}. \tag{A.2}$$

Therefore, to obtain the direct demand function system $(4a) - (4b)$ we need:

$$\alpha = [a_x b_m - a_m k]/\Delta = a_x\beta - a_m\gamma, \tag{A.3a}$$

$$\beta = b_m/\Delta, \tag{A.3b}$$

$$\gamma = k/\Delta, \tag{A.3c}$$

$$1 = b_x/\Delta, \tag{A.3d}$$

$$1 = [a_m b_x - a_x k]/\Delta = a_m - a_x\gamma. \tag{A.3e}$$

In addition, the following inequality will be used extensively:

$$\frac{1}{\Delta} = \beta - \gamma^2 > 0. \tag{A.4}$$

Finally, from $(A.3a)$, $(A.3e)$, and $(A.4)$ we have:

$$a_x = \frac{\alpha+\gamma}{\beta-\gamma^2} \quad \text{and} \quad a_m = \frac{\alpha\gamma+\beta}{\beta-\gamma^2}. \tag{A.5}$$

• Welfare Function

Equation $(8b)$ presents the government's welfare function in terms of the parameter of the direct demand system. We thus have to make use of the relationships among parameters of the direct and inverse demand systems described above. The three elements of the welfare function are:

$$CS(\tilde{P}) = (a_x - P)X(P,\tau) + (a_m - 1 - \tau)M(P,\tau) - \frac{1}{2}\left[b_x X(P,\tau)^2 + b_m M(P,\tau)^2 + 2kX(P,\tau)M(P,\tau)\right], \tag{A.6a}$$

$$\pi(P,\tau,c) = (P-c)X(P,\tau), \tag{A.6b}$$

$$R(P,\tau) = \tau M(P,\tau). \tag{A.6c}$$

Adding these three terms we get:

$$W(P,\tau,c)=(a_x-c)X(P,\tau)+(a_m-1)M(P,\tau)-\frac{b_xX(P,\tau)^2+b_mM(P,\tau)^2+2kX(P,\tau)M(P,\tau)}{2}. \quad (A.7)$$

Equation ($8b$) substitutes ($4a$) – ($4b$) and ($A.3$) – ($A.5$) into ($A.7$).

• Stationary OLE

After making $\hat{P}_c = \hat{\tau}_c = 0$ in equation (12) we have:

$$\begin{bmatrix} (\alpha+\gamma)(r-\lambda\delta)+(r-2\lambda\delta)\beta c \\ (r-2\lambda\delta)\gamma c \end{bmatrix} = \begin{bmatrix} 2\beta & -\gamma \\ \gamma & -1 \end{bmatrix} \begin{bmatrix} (r-\lambda\delta)P-\dot{P} \\ (r-\lambda\delta)\tau-\dot{\tau} \end{bmatrix}. \quad (A.8)$$

The stationary OLE is found by making $\dot{P}=0$ and $\dot{\tau}=0$. Then, P° and τ° can be easily computed from ($A.8$) using Cramer's Rule. Thus, rewriting ($A.8$) we have:

$$\begin{bmatrix} (r-\lambda\delta)P^\circ \\ (r-\lambda\delta)\tau^\circ \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{P} \\ \dot{\tau} \end{bmatrix} + \begin{bmatrix} (r-\lambda\delta) & 0 \\ 0 & (r-\lambda\delta) \end{bmatrix} \begin{bmatrix} P \\ \tau \end{bmatrix}. \quad (A.9)$$

Focusing on the homogeneous part of this dynamic system, the Routh–Hurwitz stability condition requires that [Beavis and Dobbs (1990 §5.4)]:

$$\mathrm{TR}\begin{bmatrix} (r-\lambda\delta) & 0 \\ 0 & (r-\lambda\delta) \end{bmatrix} = 2(r-\lambda\delta) < 0, \quad (A.10a)$$

and:

$$\begin{vmatrix} (r-\lambda\delta) & 0 \\ 0 & (r-\lambda\delta) \end{vmatrix} = (r-\lambda\delta)^2 > 0, \quad (A.10b)$$

from which follows that the stationary OLE is globally stable if $r < \lambda\delta$. Finally, the level of marginal cost at the stationary OLE is found making $\dot{c}=0$ in equation (6) so that:

$$\delta c = (\alpha+\gamma) - \beta P + \gamma\tau. \quad (A.11)$$

The value of c° in equation (18) is found after substituting the OLE strategies ($15a$) – ($15b$) in ($A.11$). The marginal cost in the SNE, c^N, is c^0 when $\lambda = 0$. The difference between these two stationary equilibrium levels for the state variable of the game is:

$$c^\circ - c^N = \frac{\lambda\delta(\alpha+\gamma)\beta^3}{(r-\lambda\delta)(2\beta-\gamma^2)^2\delta^2+(r-2\lambda\delta)\beta^4+(2r-3\lambda\delta)(2\beta-\gamma^2)\delta\beta^2} < 0, \quad (A.12)$$

because for any stable equilibria $r < \lambda\delta$.

• Bellman Equations

To obtain the first order conditions $(21a)-(21b)$ we need to substitute demand production and imports $(4a)-(4b)$ and the marginal cost motion equation (6) into the profit $(A.6b)$ and welfare function $(A.7)$ respectively. Bellman equations can then be written as follows:

$$rV^F(c) = \max_{P \in S^F} \left\{ \left[P - c - \lambda(\gamma - \beta)V_c^F(c)\right] \left[\alpha - \beta P + \gamma(1+\tau)\right] + \lambda\delta c V_c^F(c) \right\}, \quad (A.13a)$$

$$\begin{aligned} rV^G(c) = \max_{\tau \in S^G} \Bigg\{ & \left[\frac{\alpha+\gamma}{\beta-\gamma^2} - c - \lambda(\gamma-\beta)V_c^G(c)\right] \left[\alpha - \beta P + \gamma(1+\tau)\right] \\ & + \gamma\frac{\alpha+\gamma}{\beta-\gamma^2}[\gamma P - \tau] + \lambda\delta c V_c^G(c) - \frac{[\alpha - \beta P + \gamma(1+\tau)]^2}{2(\beta-\gamma^2)} \\ & - \frac{\beta[\gamma P - \tau]^2}{2(\beta-\gamma^2)} - \frac{\gamma[\alpha - \beta P + \gamma(1+\tau)][\gamma P - \tau]}{\beta-\gamma^2} \Bigg\}. \end{aligned} \quad (A.13b)$$

To characterize the equivalence between parameters ψ's and ϕ's, first differentiate $(23a)-(23b)$ to obtain:

$$V_c^F(c) = \psi_1^f + \psi_2^f c, \quad (A.14a)$$

$$V_c^G(c) = \psi_1^g + \psi_2^g c. \quad (A.14b)$$

After substituting these expressions and $(13a)-(13b)$ in $(22a)-(22b)$ we get the following two system of linear equations defined on the intercepts and slope of the linear Markov strategies respectively:

$$(2\beta - \gamma^2)\phi_1^f = (\alpha+\gamma) + \lambda(\gamma-\beta)[\beta\psi_1^f - \gamma^2\psi_1^g], \quad (A.15a)$$

$$(2\beta - \gamma^2)\phi_1^g = \gamma(\alpha+\gamma) + \lambda\beta\gamma(\gamma-\beta)[\psi_1^f - 2\psi_1^g], \quad (A.15b)$$

$$(2\beta - \gamma^2)\phi_2^f = (\beta-\gamma^2) + \lambda(\gamma-\beta)[\beta\psi_2^f - \gamma^2\psi_2^g], \quad (A.15c)$$

$$(2\beta - \gamma^2)\phi_2^g = -\beta\gamma + \lambda\beta\gamma(\gamma-\beta)[\psi_2^f - 2\psi_2^g]. \quad (A.15d)$$

Solving these two systems of linear equations leads to the following equivalence relations:

$$\lambda\beta\gamma(\beta-\gamma)\psi_1^f = \gamma(\alpha+\gamma) - 2\beta\gamma\phi_1^f + \gamma^2\phi_1^g, \quad (A.16a)$$

$$\lambda\beta\gamma(\beta-\gamma)\psi_1^g = -\beta\gamma\phi_1^f + \beta\phi_1^g, \quad (A.16b)$$

$$\lambda\beta\gamma(\beta-\gamma)\psi_2^f = \beta\gamma - 2\beta\gamma\phi_2^f + \gamma^2\phi_2^g, \quad (A.16c)$$

$$\lambda\beta\gamma(\beta-\gamma)\psi_2^g = \beta\gamma - \beta\gamma\phi_2^f + \beta\phi_2^g. \quad (A.16d)$$

• Riccati Equations

Substitute the proposed linear strategies (24a) – (24b) and its derivatives into (12). A stationary MPE requires $\dot{P} = \dot{\tau} = 0$ so that:

$$\begin{bmatrix} (\alpha+\gamma)(r-\lambda\delta+\gamma\lambda\phi_2^g) + (r-2\lambda\delta)\beta c \\ \\ (r-2\lambda\delta)\gamma c \end{bmatrix} = \begin{bmatrix} 2\beta & -\gamma \\ \\ \gamma & -1 \end{bmatrix} \begin{bmatrix} (r-\lambda\delta)(\phi_1^f + \phi_2^f c) \\ \\ (r-\lambda\delta)(\phi_1^g + \phi_2^g c) \end{bmatrix}$$

$$+ \begin{bmatrix} \phi_2^g & 0 \\ \\ 0 & \phi_c^f \end{bmatrix} \begin{bmatrix} \beta\gamma & -\gamma^2 \\ \\ -2\beta\gamma & \beta+\gamma^2 \end{bmatrix} \begin{bmatrix} \lambda(\phi_1^f + \phi_2^f c) \\ \\ \lambda(\phi_1^g + \phi_2^g c) \end{bmatrix}. \quad (A.17)$$

Riccati equations (25a) – (25b) are derived from here by equating the elements on c and those of the independent terms. Finally, observe that because of the block–recursive structure of these Riccati equations, once we know $\{\phi_2^f, \phi_2^g\}$ we can find $\{\phi_1^f, \phi_1^g\}$ through a linear combination implicitly defined in (25a):

$$\begin{bmatrix} (\alpha+\gamma)(r-\lambda\delta+\gamma\lambda\phi_2^g) \\ \\ 0 \end{bmatrix} = \begin{bmatrix} 2\beta(r-\lambda\delta)+\lambda\beta\gamma\phi_2^g & -\gamma(r-\lambda\delta)-\lambda\gamma^2\phi_2^g \\ \\ \gamma(r-\lambda\delta)-2\lambda\beta\gamma\phi_2^f & -(r-\lambda\delta)+\lambda(\beta+\gamma^2)\phi_2^f \end{bmatrix} \begin{bmatrix} \phi_1^f \\ \\ \phi_1^g \end{bmatrix}. \quad (A.18)$$

Each one of these Riccati equations is a particular case of the general quadratic form representation of conic sections. In particular they correspond to a hyperbola [McLenaghan and Levy (1996, §4.7.2); Miravete (2001, Appendix)]. The center of the first Riccati hyperbola is:

$$\hat{\phi}_2^f = -\frac{3(r-\lambda\delta)}{\lambda\beta} > 0, \quad \text{and} \quad \hat{\phi}_2^g = -\frac{2(r-\lambda\delta)}{\lambda\gamma} > 0. \quad (A.19a)$$

Similarly, for the second Riccati hyperbola we have:

$$\hat{\phi}_2^f = \frac{r-\lambda\delta}{\lambda(\beta+\gamma^2)} < 0, \quad \text{and} \quad \hat{\phi}_2^g = \frac{\gamma(3\beta-\gamma^2)(r-\lambda\delta)}{\lambda(\beta+\gamma^2)^2} < 0. \quad (A.19b)$$

The slopes of the asymptotes of these Riccati equations are [Miravete (2001, Appendix)]:

$$S_{11} = \frac{\gamma - \sqrt{\beta^2+\gamma^2}}{(\beta+\gamma)+\sqrt{\beta^2+\gamma^2}} < 0, \quad \text{and} \quad S_{12} = \frac{\gamma - \sqrt{\beta^2+\gamma^2}}{(\beta-\gamma)-\sqrt{\beta^2+\gamma^2}} \geq 0, \quad (A.20a)$$

$$S_{21} = 0, \quad \text{and} \quad S_{22} = \sqrt{\frac{(\beta+\gamma)^2}{(\beta+\gamma)^2+4\beta^2\gamma^2}} < 1. \quad (A.20b)$$

• Stability of Stationary MPE

Condition (26) follows from substituting equation (24) into equation (6) to obtain:

$$\dot{c} = -\lambda\left[\alpha+\gamma-\beta\phi_1^f+\gamma\phi_1^g\right] + \lambda\left[\beta\phi_2^f-\gamma\phi_2^g+\delta\right]c. \quad (A.21)$$

• **Uniqueness of Stable MPE**

1. Equation $(A.19a)$ defines the center of the hyperbola corresponding to the first Riccati equation. This center always belong to the first quadrant in the $\phi_2^f \times \phi_2^g$ space.

2. Equation $(A.19b)$ defines the center of the hyperbola corresponding to the second Riccati equation. This center always belong to the third quadrant in the $\phi_2^f \times \phi_2^g$ space.

3. Proposition 8 ensures that the slope of the stability condition is bounded from below: $\beta/\gamma < 1$.

4. Positive slopes of asymptotes are such that $S_{12} < S_{22}$. Thus we have:

$$S_{12}^{-1} = \frac{(\beta-\gamma)-\sqrt{\beta^2+\gamma^2}}{\gamma-\sqrt{\beta^2+\gamma^2}} = 1 - \frac{\beta-2\gamma}{\sqrt{\beta^2+\gamma^2}-\gamma} > 1 - \frac{\beta-2\gamma}{\beta}, \tag{A.22a}$$

$$S_{22}^{-1} = \sqrt{1+\left(\frac{2\beta\gamma}{\beta+\gamma}\right)^2} < 1 + \frac{2\beta\gamma}{\beta+\gamma} < 1 + \frac{2\beta\gamma}{\beta}. \tag{A.22b}$$

Thus, for $S_{12}^{-1} > S_{22}^{-1}$ it is sufficient that $-(\beta-2\gamma) > 2\beta\gamma$. This is equivalent to condition $\gamma > \beta/[2(1-\beta)]$ assumed in Proposition 8.

5. The major consequence of this last step is that the intersection between upward slopping asymptote of the first Riccati hyperbola and the flat asymptote of the second holds to the left of the center of the latter $(A.19b)$. This ensures the existence of a solution such that $\hat{P}_c^\star = \phi_2^f < 0$ and $\hat{\tau}_c^\star = \phi_2^g < 0$. But furthermore, it defines two non–overlapping convex cones with vertex at $(A.19b)$ –cone 1–, and $(\hat{P}_c^\star, \hat{\tau}_c^\star)$, –cone 2–, respectively. Assumptions of Proposition 8 ensure that $(\hat{P}_c^\star, \hat{\tau}_c^\star)$ is the unique stable equilibrium:

5.1 Consider the vertex $(A.19b)$ and the non–overlapping cones mentioned before. According to the Minkowski's separation theorem, there must exist a hyperplane, that passing through vertex $(A.19b)$, separates the non–overlapping convex cones [Takayama (1985, §0.B3)]. The stability condition defined in equation (26) is this separating hyperplane. Step 3 shows that its slope is larger than 1. This hyperplane is the dashed line in Figure 1.

5.2 The slope of the upward slopping asymptote of the second Riccati hyperbola is smaller than 1 according to $(A.20b)$. Thus, all three solutions in cone 1 are indeed non–stable as they fall below the hyperplane of the stability condition. Similarly, the intersection of the third quadrant, such as $\hat{P}_c^\star = \phi_2^f < 0$ and $\hat{\tau}_c^\star = \phi_2^g < 0$, defines necessarily the unique stable solution.

5.3 The value of $\delta^\star$ is found as the intersection of the stability condition with the vertical axis when this separating hyperplane passes through $(A.19b)$. Thus:

$$\delta^\star = \frac{r}{1-\gamma(\beta+\gamma^2)}. \tag{A.23}$$

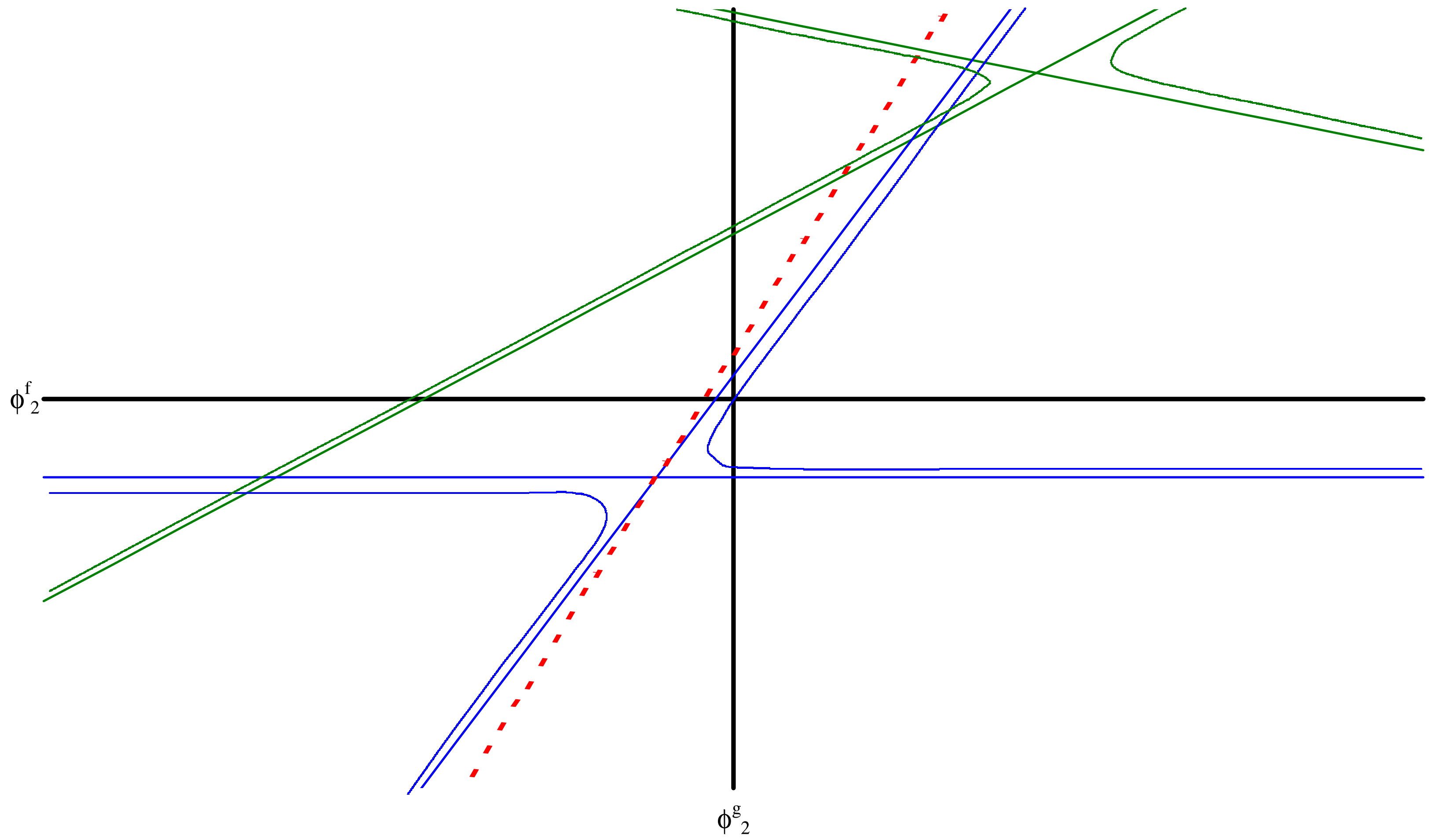

Figure 1. Riccati Equations

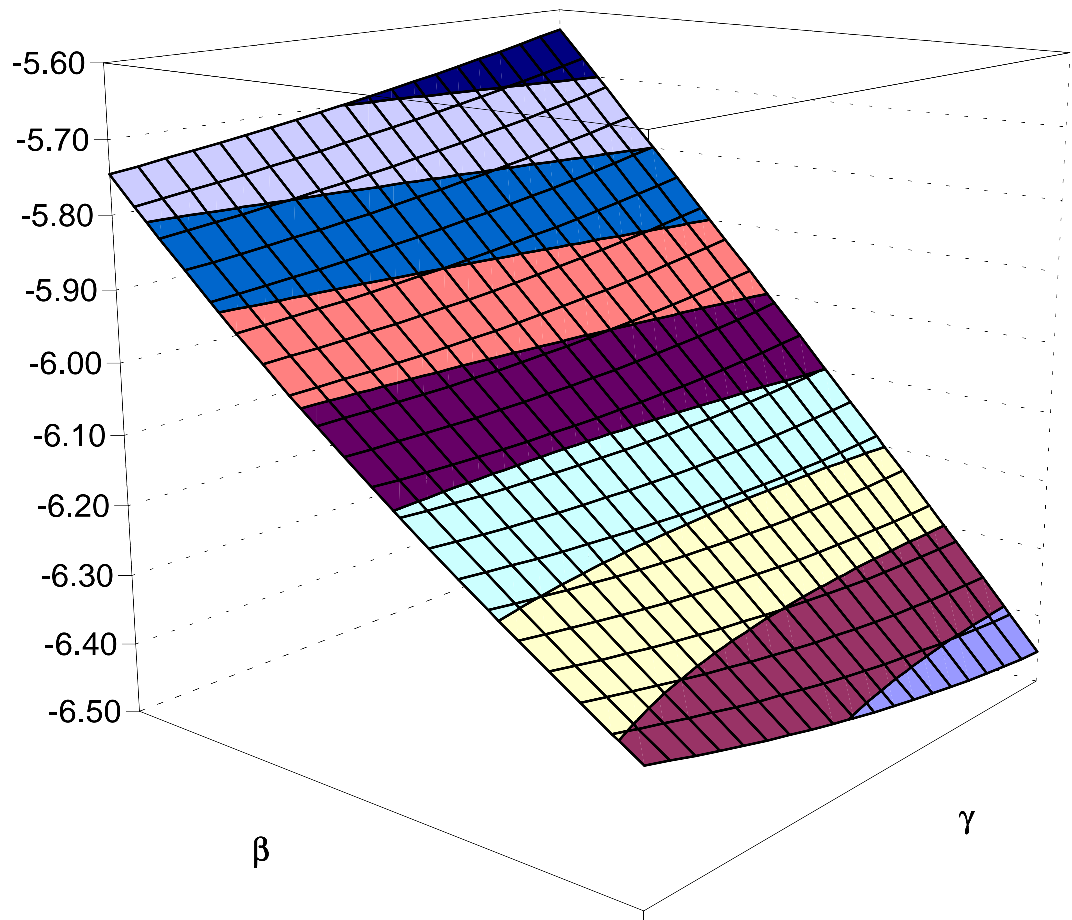

Figure 2. Comparative Statics: MPE minus OLE Prices

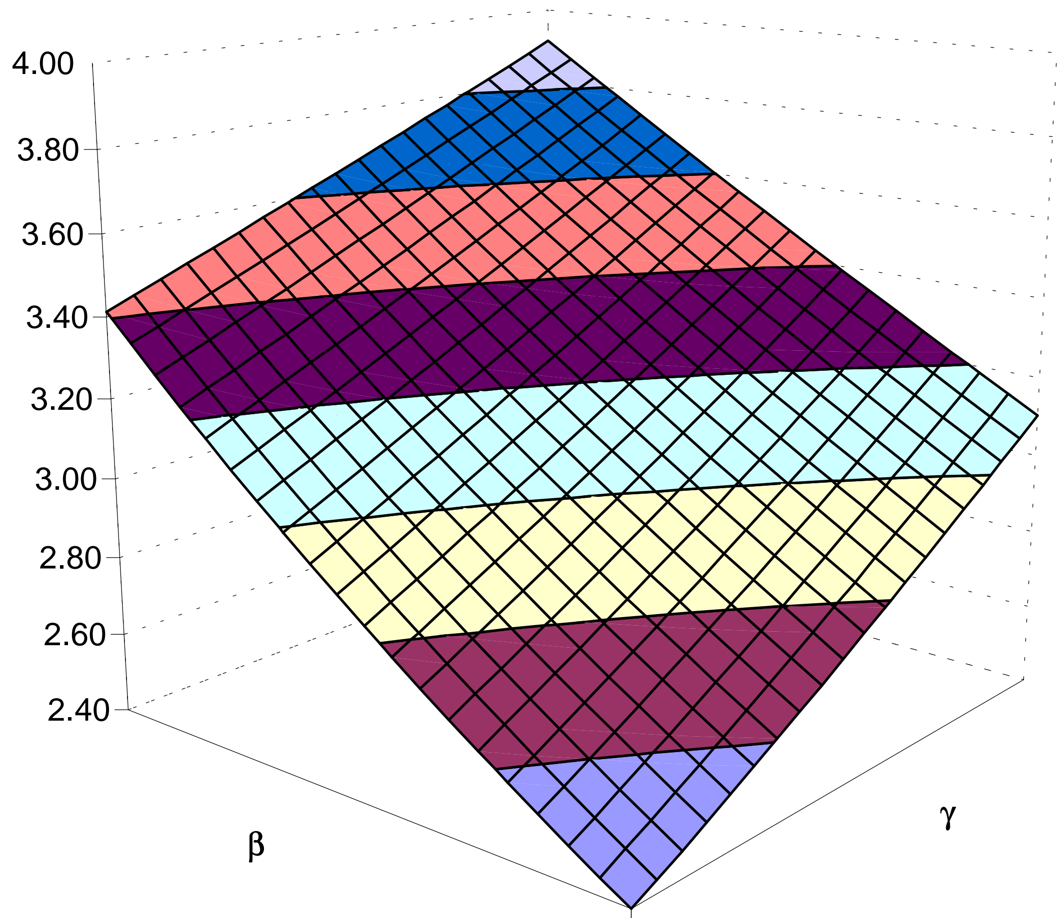

Figure 3. Comparative Statics: MPE minus OLE Tariffs

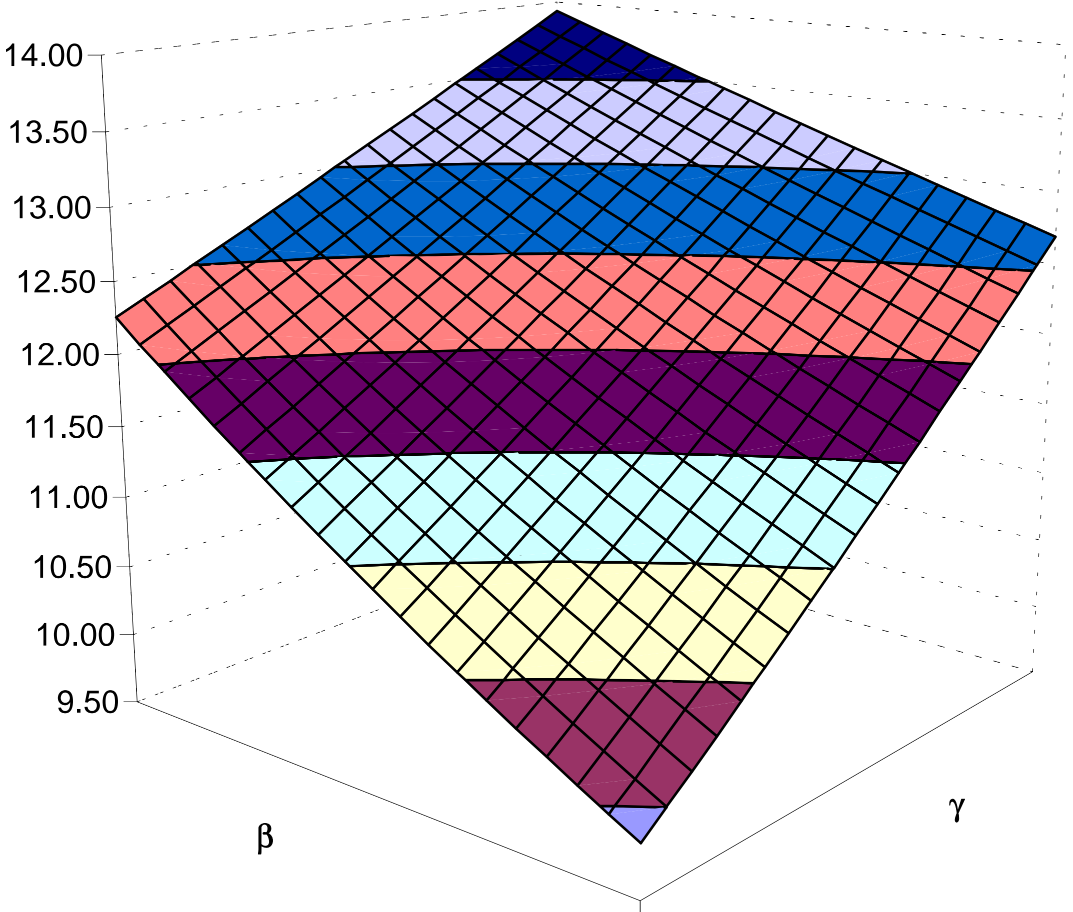

Figure 4. Comparative Statics: MPE minus OLE Marginal Costs