Estimating Demand for Local Telephone Service with Asymmetric Information and Optional Calling Plans

Eugenio J. Miravete

Appendix 2

• Derivation of the Ex–Post Tariff

The Hamiltonian of the *ex–post* pricing problem is:

$$H[V, p, \theta] = \left[\frac{(\theta_0 + \theta - p(\theta))^2}{2} + (p(\theta) - c)(\theta_0 + \theta - p(\theta)) - K - V(\theta)\right] f(\theta)$$
$$+ \delta(\theta)(\theta_0 + \theta - p(\theta)), \qquad (A.19)$$

with first order necessary conditions:

$$H_p : (p(\theta) - c)f(\theta) + \delta(\theta) = 0, \qquad (A.20a)$$
$$H_V : f(\theta) = \delta'(\theta) \; ; \; \delta(\overline{\theta}) = 0. \qquad (A.20b)$$

The transversality condition vanishes at $\overline{\theta}$ since $V'(\theta) = (\theta_0 + \theta - p(\theta)) > 0 \; \forall\theta$, so that the participation constraint is only binding at $\underline{\theta} = 0$ [Kamien and Schwartz (1991, §II.7)]. Therefore:

$$\delta(\theta) = \int_{\overline{\theta}}^{\theta} f(z)dz = F(\theta) - 1, \qquad (A.21)$$

which leads to equation (11a) in the text. The fixed fee is computed from the definition $V(\theta) = V(p(\theta), A(\theta), \theta)$, so that:

$$\hat{A}(\theta) = \frac{(\theta_0 + \theta - \hat{p}(\theta))^2}{2} - V(0) - \int_0^{\theta} (\theta_0 + z - p(z))dz. \qquad (A.22)$$

• Proof of Proposition 1

(a) For the *ex post* necessary conditions to be sufficient, it must be the case that the objective and constraint functions are concave in p and V, given that $V(\theta)$ and the Lagrange multiplier $\delta(\theta)$ are continuous, and $\delta(\theta) \geq 0$ [Kamien and Schwartz (1991, §II.3)]. It is straightforward to show that the Hessians of these functions are always singular matrices. Therefore, it suffices that the second derivative of the objective and constraint functions with respect to p be negative. The concavity of the constraint is ensured by the linearity of the demand in p. Also because of linearity of demand, the concavity of the objective function only requires that the *ex post* SCP holds: $V_{p\,\theta}(p, A, \theta) = 1 > 0$.

(b) Let $\theta > \theta'$. Incentive compatibility implies:

$$\frac{(\theta_0 + \theta - \hat{p}(\theta))^2}{2} - \hat{A}(\theta) \geq \frac{(\theta_0 + \theta - \hat{p}(\theta'))^2}{2} - \hat{A}(\theta'),$$
$$\frac{(\theta_0 + \theta' - \hat{p}(\theta'))^2}{2} - \hat{A}(\theta') \geq \frac{(\theta_0 + \theta' - \hat{p}(\theta))^2}{2} - \hat{A}(\theta). \tag{A.23}$$

Adding these two inequalities yields:

$$\begin{aligned}\int_{\theta'}^{\theta} (\theta_0 + z - \hat{p}(\theta))dz =& \frac{(\theta_0 + \theta - \hat{p}(\theta))^2}{2} - \frac{(\theta_0 + \theta' - \hat{p}(\theta))^2}{2} \\ \geq& \left[\frac{(\theta_0 + \theta - \hat{p}(\theta))^2}{2} - \hat{A}(\theta)\right] - \left[\frac{(\theta_0 + \theta' - \hat{p}(\theta'))^2}{2} - \hat{A}(\theta')\right] \\ \geq& \frac{(\theta_0 + \theta - \hat{p}(\theta'))^2}{2} - \frac{(\theta_0 + \theta' - \hat{p}(\theta'))^2}{2} = \int_{\theta'}^{\theta} \frac{\theta_0 + z - \hat{p}(\theta')}{2} dz.\end{aligned} \tag{A.24}$$

This inequality together with the single crossing property (SCP), $b > 0$, implies that $\hat{p}(\theta) \leq \hat{p}(\theta')$. Therefore, since $\hat{p}(\theta)$ is monotone, it is almost everywhere continuous and differentiable. Observe that $\hat{p}'(\theta) < 0$, *i.e.,* higher consumer types pay lower marginal tariffs. This result holds globally because of the SCP, and ensures that local maximum of the consumer tariff choice is also a global maximum. For the mechanism to be almost everywhere differentiable, it remains to be proved that the other outcome function, $\hat{A}(\theta)$, is also almost everywhere differentiable. Observe that IC also implies:

$$\frac{(\theta_0 + \theta - \hat{p}(\theta))^2 - (\theta_0 + \theta - \hat{p}(\theta'))^2}{2} \geq \hat{A}(\theta) - \hat{A}(\theta') \geq \frac{(\theta_0 + \theta' - \hat{p}(\theta))^2 - (\theta_0 + \theta' - \hat{p}(\theta'))^2}{2}. \tag{A.25}$$

Then, taking limits and using equation (5), it follows:

$$\lim_{\theta' \to \theta} \frac{(\theta_0 + \theta - \hat{p}(\theta))^2 - (\theta_0 + \theta - \hat{p}(\theta'))^2}{2(\theta - \theta')} = \lim_{\theta' \to \theta} \frac{(\theta_0 + \theta' - \hat{p}(\theta))^2 - (\theta_0 + \theta' - \hat{p}(\theta'))^2}{2(\theta - \theta')} =$$
$$\lim_{\theta' \to \theta} \frac{(\theta_0 + \theta - \hat{p}(\theta))^2 - (\theta_0 + \theta - \hat{p}(\theta'))^2}{2(\hat{p}(\theta) - \hat{p}(\theta'))} \frac{\hat{p}(\theta) - \hat{p}(\theta')}{\theta - \theta'} = -(\theta_0 + \theta - \hat{p}(\theta))\hat{p}'(\theta) = \hat{A}'(\theta). \tag{A.26}$$

Therefore higher consumer types pay higher fixed fees.

(c) For an *ex–post* consumer type θ, the optimal purchase is $\hat{x}(\theta) = x(\hat{p}(\theta), \theta)$. Consumers with higher valuations purchase larger amounts of good because if $\lambda > 0$:

$$\hat{x}'(\theta) = 1 + \lambda > 0, \tag{A.27}$$

(d) At $\theta = \overline{\theta}$, $F(\overline{\theta}) = 1$ and the second term of the marginal tariff equation (11a) equals zero, so that $\hat{p}(\overline{\theta}) = c$.

(e) A continuum of self–selecting two–part tariffs is such that each *ex–post* consumer type chooses the one which maximizes her utility. Each linear tariff is the optimal solution for only one *ex–post* consumer type, and therefore equilibrium is completely separating. A sufficient condition for a continuum of two–part tariffs to be self–selecting is that its lower envelope be concave in consumption [Faulhaber and Panzar (1977)]. Provided that $\hat{x}'(\theta) > 0$, it suffices to prove that $\hat{p}'(\theta) < 0$. But $\hat{p}'(\theta) = -\lambda < 0$ when $\lambda > 0$. ■

• Proof of Proposition 3

Parts *(a)* and *(b)* are proved similarly to Proposition 1 by substituting $(\theta_0 + \theta - \hat{p}(\theta'))^2/2 - \hat{A}(\theta')$ for $E_2[(\theta_0 + \theta_1\theta_2 - \tilde{p}(\theta_1'))^2/2 - \tilde{A}(\theta_1')]$ in order to show that $\tilde{p}'(\theta_1) < 0$ and $\tilde{A}'(\theta_1) > 0$. To prove part *c)* observe that the optimal purchase of an *ex–post* consumer type θ with an *ex–ante* type θ_1 defined in (17) is such that:

$$\frac{\partial \tilde{x}}{\partial \theta_1} = \theta_2 + \lambda_1 > 0, \tag{A.28}$$

as long as $\lambda_1 > 0$ because θ_2 is always positive. Part *d)* makes use of the distribution of θ_1 so that $F_1(1) = 1$ and $\tilde{p}(1) = c$. Finally, part *e)* the concavity of the lower envelope is ensured by $\tilde{p}'(\theta_1) = -\lambda_1 < 0$ when $\lambda_1 > 0$. ■

Appendix 3

• Description of Variables

The data includes the following set of variables. Most of them are dummy variables that take value equal to 1 for the indicated case:

AGE1	The age of the household head is between 15 and 34 years.
AGE2	The age of the household head is between 35 and 54 years.
AGE3	The age of the household head is above 54 years.
BENEFITS	The household receives some benefits such as Food Stamps, Social Security, Federal Rent Assistance, Aid to Families with Dependent Children,...

BILL	Total monthly expenditure in local telephone service.
BLACK	Ethnic group of the household head is black.
CHURCH	Household uses the telephone for charity or church work.
CITY	Household with residence in Louisville.
COLLEGE	The household head is at least a college graduate.
DINCOME	The household did not answer questions about total annual income.
HHSIZE	Number of people who regularly live in the household.
INCOME	Estimated total monthly income of the household.
MARRIED	The household head is married.
MEASURED	The household is on local measured service.
MOVED	The household moved at least once in the past five years.
ONLYMALE	The household head is male and single.
RETIRED	The household head is retired.
TEENS	Number of teenagers (between 13 and 19 years old) living in the household.
NOV	Observation for the month of November 1986.
DEC	Observation for the month of December 1986.

- **Estimation of Income**

Income was reported as a categorical variable. Categories were defined by known income ranges. In order to define a continuous variable that represents the income of households, I assume that income is distributed according to a displaced gamma distribution [Johnson, Kotz, and Balakrishnan (1995, §17)], so that the probability density function is:

$$g(x, \alpha, \beta, \gamma) = \frac{(x-\gamma)^{\alpha-1} \exp\left[-\dfrac{x-\gamma}{\beta}\right]}{\beta^{\alpha}\Gamma(\alpha)} \tag{A.29}$$

for $\alpha > 0$, $\beta > 0$, and $x > \gamma$. As the number of cases in each category (once missing values have been deleted) is known, these parameters may be estimated by maximum likelihood. The estimates of these estimates are presented in Table A1.

Table A1

Parameters	ML Estimates	t–Statistic
α	1.4224	232.805
β	2.6672	134.917
γ	-10.2914	-506.386

The number of observations is 24,132.

Then the estimated income of a household with a particular category defined by known thresholds, is computed as the expected conditional income for that category using the estimated gamma distribution. This is particularly important for the highest open ended category. Table A2 presents the annual income estimates for each reported income category.

Table A2

Category	Cases	Estimate
0–4999	2,616	1,821.97
5,000–7,499	1,998	6,070.36
7,500–9,999	1,800	8,568.68
10,000–14,999	3,366	11,794.58
15,000–19,999	3,180	16,788.45
20,000–24,999	3,120	21,784.23
25,000–34,999	3,966	27,477.29
35,000–49,999	2,586	37,667.15
$\geq$ 50,000	1,500	52,714.91

Additional References

FAULHABER, G.R. AND J.C. PANZAR (1977): "Optimal Two–Part Tariffs with Self–Selection." Bell Laboratories Economic Discussion Paper No. 74.

KAMIEN, M.I. AND N.L. SCHWARTZ (1991): *Dynamic Optimization*, 2nd edition. (Amsterdam: North–Holland).