Game Theory, Oligopoly, and Collusion

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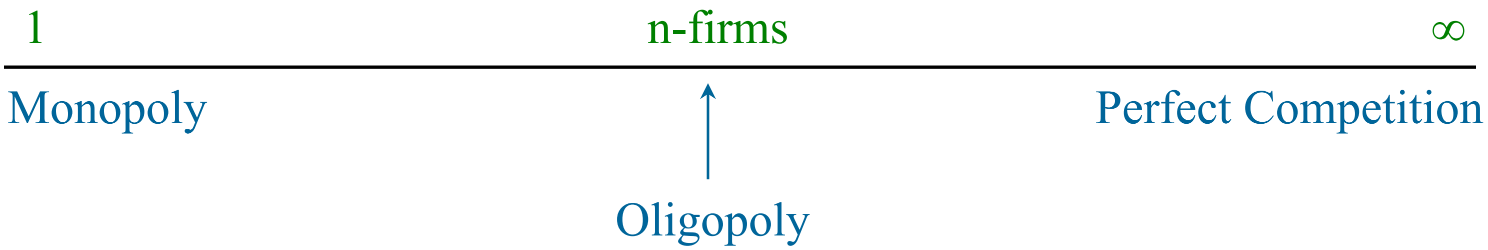
Outline

- Motivation.
- Prisoner's Dilemma.
  - Pareto dominant outcomes.
  - Dominant Strategies.
  - Nash Equilibrium.
- Battle of the Sexes.
  - Equilibrium selection:
    - Focal points.
    - Commitment.
    - Government intervention.
- Repeated Games: Collusion.

Motivation

- Strategic interaction. Firms know that their decisions will affect competitors. Thus, rational firms should react to and account for their competitors' actions.
- Tool. Game Theory:
  - Cooperative.
  - Non-cooperative.



$P(q_1 + q_2)*q_i - cq_i$

Prisoner's Dilemma: Description

- Two players (prisoners).
  - Actions: $a_i \in A_i = \{C, NC\}$.
  - Payoffs: $u_i = u_i(a_i, a_j)$.
- Features:
  - There is a Pareto dominant outcome.
  - Each player has a dominant strategy.
  - There is a unique (pure-strategy) Nash Equilibrium
- Nash Equilibrium.
  - Players choose actions that are mutual best response.
  - There is no incentive to deviate from the equilibrium outcome.
  - This is the only self-enforcing agreement possible.

Prisoner's Dilemma: Normal Form

		2	
		NC	C
1	NC	-1 , -1	-15 , 0
	C	0 , -15	-6 , -6

Prisoner's Dilemma: Pareto Dominance

	NC	C
NC	-1 , -1	-15 , 0
C	0 , -15	-6 , -6

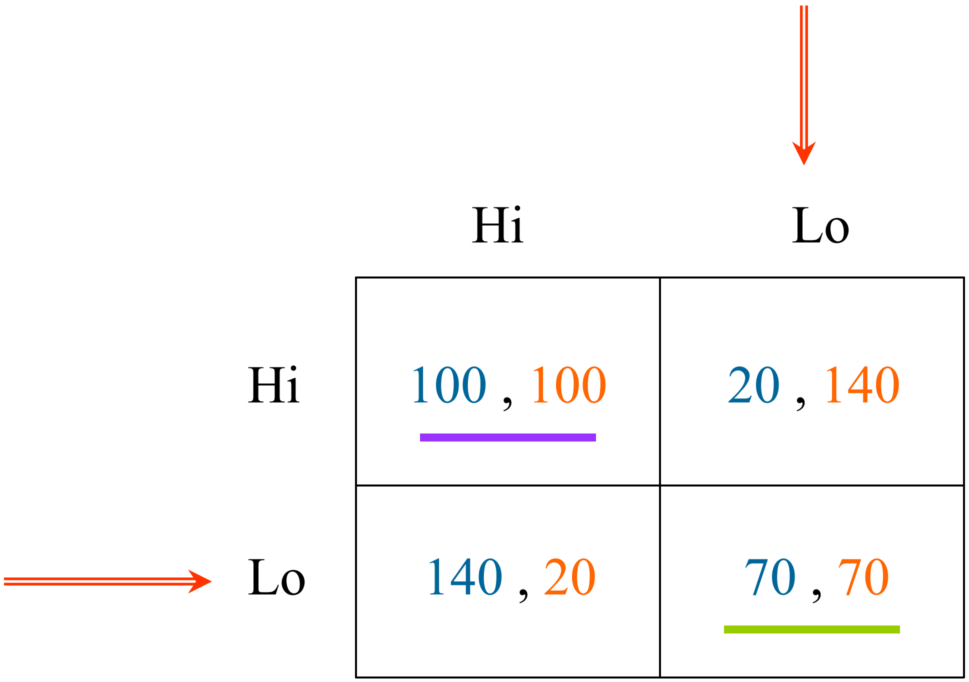
Prisoner's Dilemma: Dominant Strategies

	NC	↓ C
NC	-1 , -1	-15 , 0
→ C	0 , -15	-6 , -6

Prisoner's Dilemma: Nash Equilibrium

	NC	C
NC	-1 , -1	-15 , 0
C	0 , -15	-6 , -6

Prisoner's Dilemma: Collusion



	Hi	Lo
Hi	100 , 100	20 , 140
Lo	140 , 20	70 , 70

M.Q. 5: Cigarette Advertising

This page has been left blank on purpose. Read the motivating question above and address it before coming to class. We will discuss them and I will post the solutions after the lecture.

Battle of the Sexes: Description

- Two players (husband and wife for instance).
  - Actions: $a_i \in A_i$ = {BK, LT}.
  - Payoffs: $u_i = u_i(a_i, a_j)$.
- Features:
  - There is no Pareto dominant outcome.
  - Players do not have dominant strategies.
  - There are more than one pure-strategy Nash Equilibrium.

Battle of the Sexes: Pure Strategies

		She	
		BK	LT
Me	BK	5 , 2	1 , 1
	LT	1 , 1	2 , 5

Battle of the Sexes: Focal Points

- How to solve the multiplicity of equilibria problem?

- Example: Make two groups of five cities among the following capitals:
  - Amsterdam, Algiers, Cairo, London, Oslo, Paris, Rabat, Rome, Tripoli, and Tunis.
  - There are 252 different ways to form these two groups.
  - However, most people will probably suggested this solution:
    - Algiers, Cairo, Rabat, Tripoli, Tunis.
    - Amsterdam, London, Oslo, Paris, Rome.

- There might be important issues not explicitly considered in the description of the games!

The value of Commitment

		She	
		BK	LT
Me	BK	5 , 2	1 , 1
	~~LT~~	~~1 , 1~~	~~2 , 5~~

Note on the refrigerator: “I’ll wait for you at BK. Signed, Me.” (*Sequential Game*).

Airbus vs. Boeing (Super-Jumbo)

		B	
		In	No
A	In	-10 , -10	100 , 0
	No	0 , 100	0 , 0

What if the EU decides to subsidize Airbus by 20 if it carries out the project?

Airbus vs. Boeing (Super-Jumbo)

		B	
		In	No
A	In	-10 , -10 10 , -10	100 , 0 120 , 0
	No	0 , 100	0 , 0

Would the subsidy be necessary in the end?

Collusion: Repeated vs. One-Shot Games

	Hi	Lo
Hi	100 , 100	20 , 140
Lo	140 , 20	70 , 70

Infinitely Repeated Games

- How can we think of infinite horizons?
  - Long lived agents.
  - Uncertainty about the end of the game.

- What is a trigger strategy?

- How do we compute the payoffs of repeated games?
  - Present value of the trigger strategy.
  - Meaning of the discount value δ.

Present value of the collusion strategy

$$PV_1^c = \Pi_0 + \delta^1 \Pi_1 + \delta^2 \Pi_2 + \delta^3 \Pi_3 + \cdots = \sum_{t=0}^{n} \delta^t \Pi_t$$

$$\Pi_t = \Pi$$

$$n \to \infty$$

$$PV_1^c = \frac{\Pi}{1 - \delta}$$

Present value of the cheating strategy

$$PV_1^d = \Pi^d + \frac{\delta}{1-\delta}\Pi^N = 140 + \frac{\delta}{1-\delta}70$$

Equilibrium condition for collusion

$$\frac{\Pi^c}{1-\delta} = PV_1^c \geq PV_1^d = \Pi^d + \frac{\delta}{1-\delta}\Pi^N$$

$$\Pi^c \geq (1-\delta)\Pi^d + \delta\Pi^N$$

$$(\Pi^d - \Pi^N)\delta \geq \Pi^d - \Pi^c$$

$$\delta \geq \frac{\Pi^d - \Pi^c}{\Pi^d - \Pi^N}$$

Condition for collusion (example)

$$\frac{100}{1-\delta} = PV_1^c \geq PV_1^d = 140 + \frac{\delta}{1-\delta}70$$

$$100 \geq (1-\delta)140 + \delta 70$$

$$(140 - 70)\delta \geq 140 - 100$$

$$\delta \geq \frac{140-100}{140-70} = \frac{4}{7}$$

$$\frac{1}{1+r} \geq \frac{4}{7}$$

$$r \leq 0.75$$